Defense Spending and Economic Growth: 
A Theoretical Manifestation for Empirical Studies

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Working Paper # 12/02

February 2012

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Abstract
Defense literature is still in need of a theoretical framework in the neoclassical sense, in regard to empirical research on the relationship between defense spending and economic growth. In this respect, Dunne, Smith and Willenbockel (2005), although not without technical problems, represented a breakthrough in the field. In addition, the whole empirical literature following Mankiw, Romer and Weil (1992) is based on the unrealistic assumption that technological progress is identical across countries and constant in time. Recently, Bayraktar-Saglam and Yetkiner (2012) developed a theoretical framework that overcomes the unrealistic assumption of constant and identical rates of technological progress. In this paper, we achieve two things. First, we develop the true growth-defense model, based on Dunne, Smith and Willenbockel (2005). Second, we overcome the general weakness of constant and identical technological progress assumption in empirical growth studies by employing Bayraktar-Saglam and Yetkiner (2012) growth framework. We show that the intensity of defense spending in GDP has both positive and negative effects. In this respect, the theory supports the findings of the empirical literature, which are inconclusive in nature.

Keywords: Military Expenditure, Defense Spending, Convergence, Economic Growth

JEL Classification: H56, O30, O41, O47, O50.

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1 Introduction

In his recent work, Dunne (2010:2) states that “developing a theoretical model is important for any empirical study, but much of economic theory does not have an explicit role for military spending as a distinctive economic activity”. He continues by saying “in empirical work, the fact that there is no agreed theory of growth among economists means that there is no standard framework that military spending can be fitted into”. We agree with Dunne’s observation, and find the lack of focus on economic growth-military spending nexus surprising, for at least two main reasons. First, military expenditure has always been an important issue among economists and has always been perceived as a significant item in total GDP (=aggregate expenditure), although its share may have fallen in comparison to education or health expenditures for some countries. Hence, it is disappointing to find that the link between such a central issue and economic growth has not been sufficiently developed. Second, the (modern) growth literature has a long history since Solow (1956), with a second golden age in 1990s and 2000s after Romer (1986), Lucas (1988), Barro (1990) and Romer (1990). During this period, although many growth-related issues were thoroughly explored at the theoretical level by growth economists, it is surprising to note that the defense-related aspects of the theory did not develop to a satisfactory level.

The argument that there is no solid theory in the neoclassical sense on growth-defense nexus does not mean a total absence of work in that direction. Indeed, Dunne, Smith and Willenbockel (2005), henceforth DSW (2005), was a historical breakthrough, as it offered a neoclassical framework for empirical research in growth-defense nexus by adopting the convergence framework of the economic growth setup. However, the proposed framework had its own technical problems. In particular, we note five areas of inconsistency. First and foremost, the model is ‘unclosed’ in the sense that an item called “share of military expenditure in GDP” appears in the final good production function (as an externality), with no indication of who makes this expenditure or how it is financed. In this respect, the item is a “manna from heaven”, which is against the general equilibrium understanding. The second inconsistency results directly from the first: DSW (2005) assume that the “share of military expenditure in GDP” is function of time, without further specification, that is, leaving the “share of military expenditure in GDP” in generic form (undefined) in the model. This fact indeed makes it impossible to make the derivations that DSW (2005) did, or to draw any firm conclusion. However, DSW (2005) did just this, arguing, for example that the “share of military expenditure in GDP”
does not affect the steady state results. As we will show in an unambiguous way in the next section, this particular argument is not valid unless the growth rate of the “share of military expenditure in GDP” is zero in the steady state (normally, a share variable is always zero in the steady state; in DSW (2005), however, it is impossible to derive this result as the variable is left undefined and generic). \(^1\) The third controversial finding is the derivation of convergence equation in DSW (2005), which, given that the “share of military expenditure in GDP” is function of time, cannot be correct. Indeed, they had to define the specific behavior of the “share of military expenditure in GDP” in time in the model, which would require linearization of two equations of motion through a truncated Taylor’s expansion. Fourth, the way DSW (2005) introduce the “share of military expenditure in GDP” to the technology variable may not work in the exact way expected by DSW (2005), because a share parameter is between zero and one, and so, the share of military expenditure does indeed level down the technology. In that respect, it does have a ‘negative’ impact on technology. Finally, there are some other minor derivational errors that need to be corrected. All in all, DSW (2005) is a great advance towards a complete growth-defense framework for empirical purposes, but abovementioned inconsistencies create a serious weakness in the model and have to be removed.

Even though a solid theory on growth-defense nexus was lacking, dozens of empirical works have been produced studying the impact of military spending on economic growth. \(^2\) However, as well as an absence of a widely-accepted theoretical background for all these empirical growth-defense studies, another critical drawback, which originates from initial works of the empirical economic growth literature, was undermining all these empirical works on growth-defense nexus. Recall that two interrelated empirical research strands have emerged from the neoclassical growth theory. The first strand of

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\(^1\) Even if this assumption were correct, we could not make that particular argument. Recall that in standard convergence derivations macro variables are transformed into immeasurable variables, e.g., capital per efficient capita, for analytical tractability. The same is done in DSW (2005). However, these variables must be transformed back to measurable ones, e.g., capital per capita, to make sensible conclusions. The same applies DSW (2005). It is indeed easy to see that GDP per capita and capital per capita are a function of the “share of military expenditure in GDP”.

empirical studies aimed to determine the sources of international differences in income per capita. The second investigated whether low-income economies grow faster than the high-income ones due to diminishing marginal returns, as the neoclassical growth theory conjectures. This area of investigation quickly became dubbed convergence analysis. An intensive period of research began, especially after Mankiw, Romer and Weil (1992), which became the basic empirical framework to test for convergence. MRW (1992) estimated an augmented Solow model, which includes human capital stock in addition to physical capital stock. This model reveals that income per capita is determined by population growth, accumulation of physical capital and of human capital. Within this set up, they find strong evidence for (conditional) convergence: countries with similar technologies and rates of physical and human capital accumulation converge in income per capita. Given the Solovian set up, and the fact that technology is exogenous, it was natural for MRW (1992) to assume that the growth rate of technology does not vary across countries (in particular, it was taken as 0.02). Subsequently, following MRW (1992), most empirical studies on the convergence issue or on the sources of international differences in income per capita not only treated technology as an exogenous component but, in addition, considered it merely a component in the constant term in econometric sense. Many convergence studies estimated the convergence rate in the range 0.02 to 0.10. These studies used such equations that convergence takes place through the adjustment of the capital output ratio instead of changes in technology or its determinants. We believe as a result of this stance, a large body of empirical studies on conditional convergence has overemphasized the role of capital accumulation and, correspondingly, underestimated the role of technological change.


Henceforth, we will use MRW (1992) instead of Mankiw, Romer and Weil (1992).

Those convergence studies using panel data technique in their analysis, such as Islam (1995) and Caselli et al. (1996) overcome this problem only partially, as they allow for using individual country effects to capture the technology differences across countries. However, these studies continue to assume constant growth rate of technology for each country. E.g., Islam (1995), Caselli et al. (1996), Murthy and Chien (1997) Barro and Sala-i-Martin (1992, 2003), Nonneman and Vanhoudt (1996), and Keller and Poutvaara (2005).

This study is not alone in opposing the adoption of constant and non-varying rate of technological change across countries in convergence studies. Two good examples are Bloom et al. (2002) and Dowrick and Rogers (2002). The former objected to both the idea of identical rate of technological progress in every country and the fixed effects approach adopted by panel data versions (e.g., Islam (1995) and Caselli et al. (1996)), which allows for TFP differentials across countries but assume that these differentials persist indefinitely. The latter argued that the growth rate of technology depends on the technology gap between the leader and the follower.
A recent study by Bayraktar-Saglam and Yetkiner (2012), henceforth BS&Y (2012), argued that one major reason why the convergence literature persistently assumed constant and identical growth rate is that the Solovian framework is relatively simple, and therefore unable to differentiate technological change across economies under the exogenous technological change assumption. In this respect, BS&Y (2012) argued that the literature needs an augmented framework that allows the differentiation of economies in regard to the sources of economic growth in general, and on technological change in particular. Recall that the first-generation endogenous growth models have emphasized R&D activities as major force behind economic growth (cf., Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). Since the scale effect prediction of those models were not supported by the data (cf., Jones (1995)), this leads to the emergence of second generation endogenous growth or semi-endogenous growth models, which assume that the rate of technological progress in any particular country depends on its research intensity, that is, the proportion of labor force employed in the R&D sector and the proportion of income devoted to R&D sector.7 Hence, if the Solovian framework of convergence analysis is going to be augmented, a critical contribution would be to decompose exogenous technological change into its determinants, and then to introduce the proportion of skilled labor into that component.

To this end, BS&Y (2012) developed a framework that on the one hand allows the introduction of non-constant and non-identical technological growth rates, and on the other, retains the simplicity and appropriateness of the Solovian convergence equation. In particular, BS&Y (2012) imposed the exogenous allocation of consumption-saving tradeoff of Solow (1956) to the endogenous technological change model of Romer (1990), and called this Solovianized Romer model. The framework allows for the elimination of the assumption of the constant technology parameter in MRW-like convergence equation, since the solution of the model reveals that technological progress depends on the characteristics of the R&D sector in general and the share of R&D personnel in the labor force in particular. Moreover, the theoretical framework in which Solow (1956) meets Romer (1990) brings to forefront the role of human capital (in the final-good and R&D sectors) in convergence analysis in a more elegant

7 Examples to empirical applications of semi-endogenous growth models are Jones (2002), Kim (2008) and Kim (2011). In particular, Jones (2002) demonstrated that R&D intensity and educational attainment explain 80% of the US economic growth. Other studies in the same vein, such as Ha and Howitt (2007), Howitt (2000), Zachariadis (2004), Madsen (2008) and Ulku (2007) all underline that the rate of technological progress in one country depends on the research intensity in that country, which is, by and large, the fraction of labor force employed in the R&D sector and the fraction of income devoted to R&D sector.
way (in particular, in BS&Y (2012), human capital is not treated simply as a duplicate of physical capital, as has been the usual practice since MRW (1990)). The empirical part of the paper estimates the Solovianized Romer model for 31 OECD countries for the period 1980-2008, employing system GMM approach. The findings of BS&Y (2012), which runs the theoretical model in three different versions, can be summarized as follows:

1. All runs imply a convergence rate lower than that which is suggested by the literature.
2. The investment rate has a positive and statistically significant contribution to convergence in all runs.
3. R&D has positive and statistically significant impact on convergence in all runs regardless of the proxy BS&Y (2012) use. Both the share of labor and share of income devoted to R&D has positive and significant role on growth.
4. The role of human capital on convergence is positive in all runs, though it is statistically insignificant in some runs.

This study expands that particular framework by adding the defense sector to the model in the sense of DSW (2005). A short summary of the model is as follows: there are three private sectors, namely the final good sector, the intermediate-good sector, and R&D sector, in addition to a government. The role of government is to manage the defense sector, which hires a proportion of skilled labor in the sector, financed through income taxation of the final-good sector. The R&D sector generates new blueprints through skilled labor. We assume that the intensity of defense (which is the share of defense expenditure in total GDP and may also be interpreted as the income tax rate), is an externality in the knowledge production function of private sector. We consider this a legitimate assumption, as in practice, the defense sector positively enhances private R&D in many economies, at least in technology producing economies (a good example is the Internet). Blueprints of new knowledge are sold in the market through auctions, and firms purchased them to produce intermediate goods in a monopolistically competitive market. The final good sector exploits all intermediates to produce the final good. As knowledge stock grows, new varieties appear, and hence final output grows. Given that the saving-consumption tradeoff and the allocation of human capital among competing sectors are exogenous, we derive the long run determinants of economic growth and the convergence equation. This resembles the respective conventional equations, but allows for different technological growth rates at different times and for different countries. In addition to this, we introduce the role of defense spending in long run determinants and convergence equations
in a general equilibrium framework. In this set up, the role of the defense sector is not necessarily positive or negative. The externality effect of defense spending on R&D sector is the positive effect. On the other hand, taxation and the allocation of some skilled labor to the defense sector have negative effects in the process of long run development. Therefore, the total (net) effect can be either negative or positive, depending on the nature of country or the period. The organization of the paper is as follows: Section 2 discusses inconsistencies in the well-established DSW (1995). Section 3 presents a theoretical framework which develops a growth equation and a convergence equation for empirical use. Section 4 concludes the paper.

2 The DSW (2005) Framework

In their study, DSW (2005) proposed an “augmented Solow model”, in which they show the relationship between defense and growth. In particular, they propose the following Cobb-Douglas production function:

\[
Y(t) = (K(t))^\alpha [A(t) \cdot L(t)]^{\beta - \alpha}
\]  

(1)

where \( Y(t) \) denotes aggregate real income, \( K(t) \) is the real capital stock, \( L(t) \) is labor, and the technology parameter \( A(t) \) evolves according to:  

\[
A(t) = A_0 \cdot e^{gt} \cdot (m(t))^{\gamma}
\]  

(2)

where \( g \) is the exogenous rate of Harrod-neutral technical progress and \( m(t) \) is the share of military expenditure in GDP.  

Next, DSW (2005) defined the physical capital accumulation of the model by using Solow’s Fundamental Equation of Growth by assuming an exogenous saving rate \( s \), a constant labor force growth rate \( n \), and a given rate of capital depreciation \( \delta \).  

In particular, they described the dynamics of capital accumulation as follows:

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8 We would not call \( A(t) \) a parameter but an exogenous variable.

9 In DSW (2005), \( \gamma \) is undefined (in their notation system it is \( \theta \) instead of \( \gamma \); we reserve \( \theta \) for another use in this paper). They later made the interpretation that “\( \gamma \) represents the elasticity of steady-state income with respect to the long-run military expenditure share” (DSW, 2005, p. 458). However, we will show that this definition may not be correct. Furthermore, no hint has been made on the sign of \( \gamma \), though it plays a crucial role in the model. We presume that it is between zero and one.

10 DSW (2005) used \( d \) instead of \( \delta \) for depreciation.
\[ \hat{k} = s \cdot \tilde{k}^\alpha - (g + n + \delta)\tilde{k} \]

where \( \tilde{k} = K/[AL] \) denotes the effective capital-labor ratio and \( \alpha \) is the constant capital-output elasticity.\(^{11}\) DSW (2005) argued that the steady state level of \( \tilde{k} \) is \( \tilde{k}_{ss} = \left( \frac{s}{n + g + \delta} \right)^{\frac{1}{1-\alpha}} \) and that a permanent change in military spending does not affect the long-run steady state growth rate. Finally, DSW (2005) derive an empirically operational equation via linearizing the FEG by the truncated Taylor series expansion method. This equation is the business-as-usual convergence equation derived by forward integration. We believe that DSW (2005) model needs a number of improvements, including the way they introduce military spending into the augmented Solow model, and also in regard to some derivations and results. Firstly, military expenditure is defined as “manna from heaven” in the model. In a general equilibrium model, all spending must be financed by some source.\(^{12}\) According to DSW (2005), there is some military expenditure, \( M(t) = m(t) \cdot Y(t) \), but the source of this expenditure has not been specified. This modeling approach does not fit the idea of general equilibrium. It could have been financed e.g., by taxation, by channeling (exogenous) savings à la MRW (1992), or by assuming it to be an externality in the form of a technological spillover. Unfortunately, none of these is undertaken or assumed in DSW (2005). Since it was defined as expenditure in the model economy, the way it is financed should have been explicitly shown. In this respect, the model is "unclosed"; the way military expenditure is defined in the model is nothing but an assumption, produced without rationale or legitimization.

Secondly, DSW (2005) made the following interpretation immediately after their specification of aggregate production function and technology (equations (1) and (2)):

"According to this specification, a permanent change in \( m(t) \) does not affect the long-run steady state growth rate, but has potentially a permanent level effect on per-capita income along the steady-state growth path and affects transitory growth rates along the path to the new steady-state equilibrium" (DSW, 2005, p. 457).

\(^{11}\) \( \hat{k} \) in our model is equivalent to \( k_e \) in DSW (2005).

\(^{12}\) Solow is also a general equilibrium model with its own characteristics, cf. Acemoglu, (2008).
We believe the statement that a permanent change in \( m(t) \) does not affect the long-run steady state growth rate is a serious oversimplification. We can show this in several ways. One way to see it is via log differentiating the aggregate production function:

\[
Y(t) = (K(t))^{\alpha} [A(t)L(t)]^{1-\alpha} \\
\ln[Y(t)] = \alpha \ln[K(t)] + (1-\alpha) \ln[A(t)] + (1-\alpha) \ln[L(t)] \\
\ln[Y(t)] = \alpha \ln[K(t)] + (1-\alpha) \ln[A(t)] + g(t) + \gamma \ln[m(t)] + (1-\alpha) \ln[L(t)] \\
\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + (1-\alpha) g + (1-\alpha) \gamma \frac{\dot{m}}{m} + (1-\alpha) n
\]

Now imagine that the model is in the steady state. To make it absolutely clear that we are at the steady state, let us re-write the equation above at steady state:

\[
\frac{\dot{Y}_{ss}}{Y_{ss}} = \alpha \frac{\dot{K}_{ss}}{K_{ss}} + (1-\alpha) g + (1-\alpha) \gamma \frac{\dot{m}_{ss}}{m_{ss}} + (1-\alpha) n
\]

In the conventional Solow model (that there is no \( m(t) \)), physical capital would grow at the rate of output, and hence the steady state growth rate would have been \( \frac{\dot{Y}_{ss}}{Y_{ss}} = g + n \). However, in the differential equation above we cannot make such a statement, even if capital grows at the rate of output in the steady state, because there is no information in the model on whether \( \frac{\dot{m}_{ss}}{m_{ss}} = 0 \) in the steady state or not. Hence, military spending is not only effective on the growth rate during the transitional period but also at the steady state, unless \( \frac{\dot{m}_{ss}}{m_{ss}} = 0 \).

We know from growth theory that share variables are expected to converge to a constant in the long-run. However, as DSW (2005) made no specification on the behavior of military spending through time, it is impossible to make a particular statement on the steady state behavior of share of military spending to GDP.

Third, there is a typo in DSW (2005) in the derivation of fundamental equation of growth expressed in per efficient capita. We can easily show this by starting from aggregate level \( \dot{K} = s \cdot Y - \delta \cdot K \) and dividing both sides of it by efficiency labor:

\[
\frac{\dot{K}}{AL} = s \frac{(K(t))^{\alpha} [A(t)L(t)]^{1-\alpha}}{AL} - \delta \frac{K}{AL} \Rightarrow \frac{\dot{K}}{AL} = s \cdot \tilde{k}^{\alpha} - \delta \cdot \tilde{k}.
\]

10
The critical step here is transformation of \( \frac{\dot{K}}{AL} \) into \( \tilde{k} \). Note that time derivative of \( \frac{K}{AL} \) is \( \frac{d}{dt} \left( \frac{K}{AL} \right) = \frac{\dot{K}}{AL} - \frac{\tilde{k}}{A} \frac{\dot{L}}{L} \). Given the assumptions made by DSW (2005) on labor growth rate and exogenous technology, namely \( \frac{\dot{L}}{L} = n \) and \( \frac{\dot{A}}{A} = g + \gamma \frac{\dot{m}}{m} \), the differential equation turns out to be

\[
\dot{k} = s \cdot \tilde{k}^\alpha - (n + g + \gamma \cdot \dot{m} + \delta) \tilde{k} \tag{4}
\]

where a hat on top of a variable is the growth rate of that variable. Clearly, the true definition of capital per efficient capita accumulation function is different from the way DSW (2005) define it (see equation (8) in DSW (2005)). One important implication of this inconsistency is that the convergence equation derived by DSW (2005) cannot be correct, as convergence equation takes into account transitional period and not steady state.

Lastly, let us look at the steady state value of capital per efficiency labor. For this, we first need to prove how capital per efficiency labor behaves in the steady state. Suppose now that the system is in steady state. In this case, equation (4) can be written as \( \frac{\ddot{k}}{k_{ss}} = s \cdot \tilde{k}_{ss}^{\alpha-1} - (n + g + \gamma \cdot \dot{m}_{ss} + \delta) \). Taking time derivative of both sides, we obtain

\[
\frac{d}{dt} \left( \frac{\ddot{k}_{ss}}{k_{ss}} \right) = 0 = (\alpha - 1) \cdot s \cdot \tilde{k}_{ss}^{\alpha-2} \cdot \tilde{k}_{ss} - \gamma \cdot \frac{d}{dt} (\dot{m}_{ss}) \tag{5}
\]

We know from the literature that share variables do not grow in the steady state. In that respect, it must be true that \( \frac{d}{dt} (\dot{m}_{ss}) = 0 \) (that is, the growth rate of share of military expenditure is constant in the steady state). Hence, the steady state value of capital per efficient capita is

\[
\tilde{k}_{ss} = \left( \frac{s}{n + g + \gamma \cdot \dot{m}_{ss} + \delta} \right)^{\frac{1}{\alpha-1}} \tag{5}
\]

Equation (5) indicates clearly that the growth rate of military spending plays a role in the steady-state value of capital per efficient labor. Again, we obtain the result of DSW (2005) of equation (9) in their paper, only if \( \dot{m}_{ss} = 0 \) (that is,
growth rate of share of military expenditure is zero in the steady state, not an arbitrary constant).

More than this, as capital per efficient labor is immeasurable; we need to go back to per capita value. Hence, recalling that $\bar{k} = K/[AL]$, per capita physical capital must be $k = A \cdot \bar{k}$. Hence,

$$k_{ss} = \left( \frac{s}{n + g + \gamma \cdot \hat{m}_{ss} + \delta} \right)^{1-\alpha} A_0 \cdot e^{\gamma \cdot \varphi} \cdot m_{ss}^\varphi$$

(6)

What is problematic in (6) is that we have no idea on $m_{ss}$. In that respect, $k_{ss}$ and hence $y_{ss}$ are indeed unknown. Furthermore, (6) implies that the steady state growth rate of capital per capita, and hence income per capita, is function of military share as well as exogenous technological growth rate:

$$\hat{k}_{ss} = g + \gamma \cdot \hat{m}_{ss}$$

(7)

as long as $m_{ss}$ is not constant. Hence, the statement made by DSW (2005) that a permanent change in $m$ does not affect the long-run steady state growth rate is an oversimplification, given initial assumptions.

Finally, assuming that the technology variable $A(t)$ evolves according to $A(t) = A_0 \cdot e^{\gamma \cdot (m(t))^\varphi}$ does indeed mean that military expenditure scales down and not up the variable, as $m(t)$ is a share parameter between zero and one. DSW (2005), on the other hand, have the following comment on their definition of technology:

“According to this specification, a permanent change in $m$ does not affect the long-run steady-state growth rate, but has potentially a permanent level effect on per-capita income along the steady-state growth path and affects transitory growth rates along the path to the new steady-state equilibrium” (DSW, 2005, p. 457).

This comment does not clarify whether DSW (2005) perceived rightly the scaling-down nature of military expenditure on technology or not. We believe that this is a critical ambiguity in the model, given the major role of the military expenditure.
3 The Solovianized Romer Model

As the Romer (1990) model is now widely known we will be as compact as possible in its presentation. Following Romer (1990), we assume that the production technology has an additively separable characteristic:

\[ Y = H_Y^{\frac{1}{1-\alpha}} \sum_{i=1}^{A(t)} X_i^\alpha \quad 0 < \alpha < 1 \]  

(8)

where \( Y \) is final good (GDP), \( H_Y \) is the number of human capital used in final good production, \( 1-\alpha \) is the respective production elasticity of that human capital, \( X_i \) are intermediate goods (varieties), and \( A(t) \) is the number of intermediate goods at time \( t \).\(^{13}\) It should be noted that the form of (1) is Cobb-Douglas, \( Y = H_Y^{\frac{1-\alpha}{\alpha}} (X^e)^{\frac{1}{1-\alpha}} \), for \( X^e = \left( \sum_{i=1}^{A(t)} X_i^a \right)^\frac{1}{\alpha} \), where \( X^e \) may be called efficient capital stock. We assume that human capital is allocated among three sectors: the final-good, R&D, and the defense, that is,

\[ H_Y = \theta_Y \cdot \bar{H}, \quad H_{R&D} = \theta_{R&D} \cdot \bar{H}, \text{ and } H_M = \theta_M \cdot \bar{H} \]  

(9)

where \( H_Y \) is the human capital employed by the final-good, \( H_{R&D} \) is the human capital employed by the R&D sector, \( H_M \) is number of human capital employed by the military sector; \( \theta_Y, \theta_{R&D}, \text{ and } \theta_M \) are the shares of human capital in the respective sector, and \( \bar{H} \) is the stock of human capital, which is constant. Note that \( \theta_Y + \theta_{R&D} + \theta_M = 1 \) at all times. In the original Romer model, the allocation of human capital between competing sectors is endogenous. Below, considering our aim of deriving an empirically usable convergence equation, we will assume that the tradeoff is exogenous to the model.\(^{14}\) Hence, \( \theta_Y, \theta_{R&D} \text{ and } \theta_M \) are constant.

\(^{13}\) We purposefully refrained from defining unskilled labor as the third argument in the production function, à la Romer (1990), to keep derivations simple. As a drawback of this simplification, we will end up with a convergence equation in which some coefficients are unitary. Interested readers may refer to Annex B in BS&Y (2011) to observe that unit values of these coefficients should not be taken literally, and that this is due to a preference for a simplified model.\(^{14}\)
Defense Sector

We assume that the military sector is run by the government and financed through proportional income taxation. At each time $t$, the government charges a constant tax on the output (=GDP) and uses this revenue to cover the costs of military spending, which is simply the cost of human capital hired in that sector. We assume that the government follows a balanced budget, which implies

$$w_M \cdot H_M = M = \tau_M \cdot Y$$

(10)

In (10), $w_M$ is the wage cost of per unit skilled labor hired by the government in the defense sector, and $\tau_M$ is the fixed proportional tax rate. Note that the alternative interpretation of $\tau_M$ is the share of military spending in GDP, which corresponds to $m(t)$ in DSW (2005). Also note that we may express equation (10) as in intensive form, as $w_M \cdot \theta_M = \tau_M \cdot y$, where $y$ is output per skilled labor.

Final Good Sector

We assume that there is perfect competition in the final-good sector and we take final output to be the numéraire. Hence, the profit equation is

$$\Pi_y = \bar{H} \cdot \left[ (1 - \tau_M) \cdot \theta_Y^{1-\alpha} \sum_{i=1}^{A(t)} x_i^\alpha - w_Y \cdot \theta_Y - \sum_{i=1}^{A(t)} p_i \cdot x_i \right]$$

(11)

where $x_i$ are intermediate goods in per capita, $w_Y$ is the real wage rate for the skilled labor in final good sector, and $p_i$ is the user cost of intermediate-good $i$. The level of demand for each intermediate and human capital (employed at final-good production) follows directly from the first order profit maximization conditions:

$$\frac{\partial \Pi_y}{\partial H_Y} = (1 - \alpha) \cdot (1 - \tau_M) \cdot \theta_Y^{1-\alpha} \cdot \sum_{i=1}^{A(t)} x_i^\alpha - w_Y = 0$$

(12a)

$$\frac{\partial \Pi_y}{\partial X_i} = \alpha \cdot (1 - \tau_M) \cdot \theta_Y^{1-\alpha} \cdot x_i^{\alpha-1} - p_i = 0 \quad \forall i$$

(12b)

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14 Interested readers may refer to Annex A in BS&Y (2012) to see how results change when labor allocation is endogenous in the original model.
The above-given first-order profit maximization with respect to $H_y$ and $X_i$ are inverse demand functions for human capital (employed at final-good production) and any individual intermediate $i$.

**Intermediate-good Sector**

We assume that the intermediate-good producing sectors only use ‘raw’ capital in order to produce an intermediate good: $K_i = X_i$ (or $k_i = x_i$ in per capita terms), where $K_i$ measures the total amount of raw capital going into intermediate good of type $i$. Raw capital can be rented at the real rate of interest $r$ plus depreciation $\delta$: $r' = r + \delta$. Hence, $r'$ is the rental rate of capital. We assume that each intermediate-good producer has monopoly power right over the production and sale of the good $X_i$, as the patent (the blueprint) of the product belongs to it. Hence, the seller of the intermediate good faces a downward-sloping demand curve (cf., equation (12)). Therefore, the profit maximizing price for the intermediate good $i$ is obtained as $p_i = p = \frac{r'}{\alpha}$, which underlines that price is identical across intermediates. Note that substitution of price information in equation (12b) reveals that

$$
\frac{\alpha^2 \cdot (1 - \tau_M)}{r'}
$$

across varieties. Identical prices and quantities across intermediates naturally imply that profits are identical across intermediates as well, $\pi_i = \frac{\Pi_i}{H} = (1 - \alpha) \cdot \alpha \cdot (1 - \tau_M) \cdot \theta_y^{(i)} \cdot x^\alpha$. Given our findings that price, quantity and profit are identical across intermediates, it must be true that

$$
K = \sum_{i=1}^{A(t)} K_i = \sum_{i=1}^{A(t)} X_i = X \cdot A \text{ or } k = \sum_{i=1}^{A(t)} k_i = \sum_{i=1}^{A(t)} x_i = x \cdot A. \text{ Using this information in (8) yields}
$$

$$
y = \theta_y^{(i)} \cdot x^\alpha \cdot A \text{ or } y = \theta_y^{(i)} \cdot k^\alpha \cdot A^{(i)}
$$

(13)

Note from above derivations that $x = x(r)$ for a given $\theta_y$. Below, we will show that $r$ is constant at steady state. Hence, $y$ and $k$ grow at the rate of $A$ at steady state.
**R&D Sector**

À la Romer (1990), we will define the knowledge production function as

\[ \dot{A} = \eta \cdot H_{R&D} \cdot (1 + \tau_M) \cdot A. \]

We argue that the intensity of research in the defense sector positively augments the externality effect of the existing stock of knowledge. It should be noted that the defense sector presents a tradeoff from the viewpoint of the R&D sector. On the one hand, this sector augments the growth rate of knowledge. On the other, the higher the share of human capital in defense sector, the lower the human capital available for final good production and private R&D. Given (9), the growth rate of knowledge accumulation, \( \dot{A} = \eta \cdot \theta_{R&D} \cdot H \cdot (1 + \tau_M) \equiv g \), is exogenous, as in MRW (1992).\(^{15}\)

The difference is that we now know what its components are.

Given the assumption that R&D sector is perfectly competitive and that the knowledge production function is known, the value of the \( i \)th patent may easily be determined as

\[ V_{R&D,i}(t) = \frac{\bar{H} \cdot \pi(t)}{r(t)}, \]

which is an arbitrage rule stating that the return out of investing an amount equal to the value of patent in the ‘financial market’ at time \( t \), \( r(t) \cdot V_{R&D,i}(t) \), must be equal to profit \( L \cdot \pi(t) \) derived from that patent at time \( t \). The arbitrage rule is valid as long as per capita profit is constant. Recall that \( \pi = \pi(r, \theta_e) \). Hence, as long as the real rate of interest \( r \) is constant, which is true at steady state, the arbitrage condition would be as derived above. Finally, note that using the arbitrage rule in the first-order profit maximization condition in R&D sector, \( V_{R&D} \cdot \eta \cdot A = w_{R&D} \), implies

\[ \frac{\bar{H} \cdot \pi(t)}{r(t)} \cdot \eta \cdot A = w_{R&D}, \]

where \( w_{R&D} \) is the real wage rate in the R&D sector.

**Consumption-saving Tradeoff**

We assume that the consumption-saving tradeoff is exogenous, à la Solow (1956). This assumption allows a great simplification of Romer (1990), without losing the main deriving forces of the role of technological progress on economic growth. We assume that capital accumulation is led by:

\[ \dot{K} = s \cdot (1 - \tau_M) \cdot Y - \delta \cdot K \]  

(14)

where \( K \) is capital, \( s \) is the exogenous saving (investment) rate, \( Y \) is output and \( \delta \) is depreciation rate of capital.

---

\(^{15}\) A hat on top of a variable indicates the growth rate.
Long-run Equilibrium

Recall that knowledge growth rate is \( \dot{A} = \eta \cdot \theta_{R&D} \cdot H \cdot (1 + \tau_M) \). Let us now determine the steady state value of unknowns of the model. To this end, first, by using the capital accumulation function in (6), we can show that

\[
\dot{k} = s \cdot (1 - \tau_M) \cdot \delta^{1-\alpha} \cdot \dot{k}^{\alpha-1} - (\delta + g)
\]

where a tilde on top of a variable defines per efficient capita, e.g., \( \tilde{k} = \frac{K}{A \cdot H} \). It is well-known that capital per efficient capita does not grow at the steady state. Hence, the steady state value of capital per efficient capita is

\[
\tilde{k}_{ss} = \theta_Y \cdot \left( \frac{s \cdot (1 - \tau_M)}{\delta + g} \right)^\frac{1}{1-\alpha}
\]

and subsequently \( \tilde{y}_{ss} = \theta_Y \cdot \left( \frac{s \cdot (1 - \tau_M)}{\delta + g} \right)^\frac{\alpha}{1-\alpha} \).

Recall that, by construction, \( \tilde{k}_{ss} \) found above may be interpreted as the supply of capital per efficient capita in the model. We also need to define the demand for capital. Recall that we had already indicated that \( k = \sum_{i=1}^{A(t)} k_i = \sum_{i=1}^{A(t)} x_i = x \cdot A \), which implies \( \tilde{k} = x \). Hence, at steady state (capital accumulation still has transitional dynamics),

\[
\left( \frac{s \cdot (1 - \tau_M)}{\delta + g} \right)^\frac{1}{1-\alpha} \cdot \theta_Y = \theta_Y \cdot \left( \frac{\alpha \cdot (1 - \tau_M)}{r'_{ss}} \right)^\frac{1}{1-\alpha} \Rightarrow r'_{ss} = \frac{\alpha^2 (\delta + g)}{s}.
\]

Note that \( r'_{ss} \) responds negatively to an increase in saving rate and positively to the exogenous growth rate. The rest of the model can be solved through substitution. It is easy to show that the real rate of interest, the price of intermediate-good, the quantity of each intermediate-good and the profit for each intermediate are all constant at the steady state. As a final note, \( \hat{y}_{ss} = \hat{k}_{ss} = \dot{A} \equiv g \). We observe that the growth rate of GDP increases if (i) the productivity of R&D sector increases, (ii) the size or the share of human capital increases or the share of human capital allocated to defense decreases, (iii) the intensity of defense expenditure increases. GDP per efficient capita, on other hand, is positively associated with the exogenous saving rate and negatively associated with the intensity of defense spending.
2.1 Sources of International Differences in Income

Recall that we found the steady state value of output per efficient capita as

$$\tilde{y}_{ss} = \theta_t \left( \frac{s \cdot (1 - \tau_M)}{\delta + g} \right)^{1-\alpha}. $$

By taking its natural log, and then making a few algebraic transformations, we can show that

$$ \ln y_{ss} = a + g \cdot t + \ln[\theta_t] + \frac{\alpha}{1 - \alpha} \ln[s] + \frac{\alpha}{1 - \alpha} \ln[1 - \tau_M] - \frac{\alpha}{1 - \alpha} \ln[\delta + g] + \varepsilon $$

(16)

where \( \ln[A(0)] = a + \varepsilon \) and \( g = \eta \cdot \theta_{R&D} \cdot \bar{H} \cdot (1 + \tau_M). \) Equation (16) may be used for studying the contribution of defense sector to the differences in income per capita between economies in the long-run, à la MRW (1992).\(^{16}\)

The main contribution of equation (16) is that it reveals the negative effects of the defense sector on the level of development in the long run. In particular, it shows the negative effects on long run development of income taxation for financing the defense sector and the allocation of skilled labor in the defense sector. In addition, the following may be considered as added value: (i) the model decomposes the exogenous growth rate into its components, (ii) human capital is incorporated into the model through a general equilibrium modeling approach, (iii) the three-sector structure of the model has obvious advantages over the one-sector Solow framework in terms of its flexibility, and therefore it has the potential for extending the framework for further research questions.

No Spillover Effect

Finally, we would like to extend the above analysis to a certain type of economies that incur military expenditure without any spillover effect on the private R&D sector. In this group we can consider MENA countries, in which all taxation is used for supplying military personnel and purchasing arms and equipment. In this particular group of countries, one may assume that no human capital is employed in the defense sector, that is, \( \theta_t + \theta_{R&D} = 1 \) and that at each time \( t \), the government charges a constant tax on the output (=GDP) to finance the salaries of military personnel (soldiers, etc) and other expenses:

\(^{16}\) Recall that in the standard Solow set-up, the steady state value of output per efficient capita is obtained as \( \tilde{y}_{ss} = \left( \frac{s}{\delta + g} \right)^{1-\alpha} \) for \( Y = K^\mu (A \cdot \bar{H})^{1-\alpha} \) and \( A = A(0) \cdot e^{\varepsilon t}. \) Hence, the determinants of economic growth are found as \( \ln y_{ss} = a + g \cdot t + \frac{\alpha}{1 - \alpha} \ln[s] - \frac{\alpha}{1 - \alpha} \ln[\delta + g] + \varepsilon, \) where \( \ln[A(0)] = a + \varepsilon. \) Note that the determinants of \( g \) and \( A(0) \) are undefined in that case.
In (17), \( w_M \) is the wage cost of per unit unskilled labor hired by the government in the defense sector and \( L \) is the stock of military personnel (no skills but military skills). If we resolve the whole model under these assumptions, we find again (16), but with the following modifications:

\[ g = \eta \cdot \theta_{r&D} \cdot \bar{H} \quad \text{and} \quad \theta_y + \theta_{r&D} = 1. \]

Again theory suggests that the intensity of defense spending has negative effects on long run economic growth.

2.2 Convergence

The defense sector and the components of technological progress may also be important in the transition to the steady state. In particular, it is interesting to determine whether the intensity of defense has any significant role in convergence of economies. As convergence derivations are well known, we will keep this part to a minimum. Recall that capital accumulation function is given by (15). We first need to express (15) in terms of \( \bar{y} \), as it is more convenient to work with GDP per effective capita for derivations. To this end, through using log differentiated production function, that is, \( \hat{y} = \alpha \cdot \hat{k} \) (recall that \( \hat{y} = \hat{y}/\bar{y} \) and \( \hat{k} = \hat{k}/\bar{k} \)), and after simple arithmetic operations, we may re-express (7) in terms of \( \bar{y} \):

\[
\frac{d \ln(\bar{y})}{dt} = \alpha \left[ s \cdot (1 - \tau_M) \cdot \theta_y^\alpha \cdot e^{\frac{a-1}{a} \ln(\bar{y})} - (\delta + g) \right] \equiv \phi(\ln(\bar{y})) \quad (18)
\]

The differential equation in (18) is not linear. Through log-linearization, we find that

\[
\frac{d \ln(\bar{y})}{dt} \approx -(1 - \alpha)(\delta + g) \left[ \ln(\bar{y}) - \ln(\bar{y}_{sl}) \right] \quad (19)
\]
Let us now define \( \nu = (1-\alpha)(\delta + g) \). Hence, the speed that an economy converges to its own steady state is \( cr = \frac{d(\tilde{y})}{d\ln(\tilde{y})} \approx -\nu \). The solution of the linearized differential equation in (19) yields

\[
\text{LHS of (20) is the growth rate of per capita income relative to initial level. Determinants of this change in level of per capita take place on the RHS. There are two constants in } \beta_0: \text{ total growth rate between the initial time and ending time, } g \cdot t, \text{ and } (1-e^{-\nu t}) \cdot \text{Ln}(A(0)). \beta_1 \text{ is the coefficient of initial level of income per capita. Notably, this coefficient is negative, which is consistent with the convergence idea. Coefficient } \beta_2 \text{ captures the contribution of human capital on convergence. The higher the share of human capital used in final good production, the higher is the growth rate. Coefficients } \beta_3, \beta_4 \text{ and } \beta_5 \text{ show the contribution of the investment rate, the intensity of defense sector and the effective depreciation on convergence, respectively. Clearly, the very existence of defense sector results in a range of impacts that are greater than its direct effects. The defense sector affects convergence analysis through its effect on the rate of technological progress and on the share of human capital allocated to final good production. The equation also overcomes two drawbacks of the existing convergence literature, cf., MRW (1992). First, } g \text{ has been decomposed into its components (see equation (12) in BS&Y (2012) for an application of the convergence equation). Contrary to the practice of many previous studies, it is theoretically consistent not to take the rate of technological progress as being same and constant across economies. The second improvement is that the model incorporates human capital into the model in a more elegant way in the convergence equation. Another potential}

\[
\text{Just to understand what (19) actually implies, suppose that } \text{Ln}(\tilde{y}_t) < \text{Ln}(\tilde{y}_{ss}). \text{ As } \text{Ln}(\tilde{y}) - \text{Ln}(\tilde{y}_{ss}) \text{ is negative, } -\nu \cdot \left[ \text{Ln}(\tilde{y}_t) - \text{Ln}(\tilde{y}_{ss}) \right] \text{ would be positive. More than this, the higher the difference between } \text{Ln}(\tilde{y}) \text{ and } \text{Ln}(\tilde{y}_{ss}), \text{ the higher would be } -\nu \cdot \left[ \text{Ln}(\tilde{y}_t) - \text{Ln}(\tilde{y}_{ss}) \right]. \text{ In this case, an economy further away from its steady state would have a higher growth rate.}
\]
value added over the existing literature is that, by using the competitive equilibrium approach, the three-sector nature of the model allows further extensions in several directions.

_No Spillover Effect_

Let us re-examine the results of our convergence results under the no spillover assumption. Recall we had assumed that no human capital is employed in the defense sector, that is, \( \theta_Y + \theta_{R&D} = 1 \), and that the government charges a constant tax on output to finance the salaries of unskilled labor (e.g., military personnel), \( w_M \cdot L \equiv M = \tau_M \cdot Y \). In that case, again we find (20), but with the following modifications: \( g = \eta \cdot \theta_{R&D} \cdot \bar{H} \) and \( \theta_Y + \theta_{R&D} = 1 \). Hence, the intensity of defense spending does only have a negative effect on economic growth during the transitional period according to theory.

4 _Concluding Remarks_

We still lack an empirically consistent theoretical framework, in the neoclassical sense, on the relationship between defense spending and economic growth. DSW (2005) was a historical breakthrough in that direction, although not without its own technical problems. In addition, the overall convergence literature based on MRW (1992) suffers from the unrealistic assumption that all countries have an identical and unchanging rate of technological progress. Recently, BS&Y (2011) developed a theoretical framework that overcomes this unrealistic assumption. The current study introduced defense spending to the BS&Y (2011) model, in the sense of DSW (2005), but removed the inconsistencies of DSW (2005). We show that the intensity of defense spending in GDP has both positive and negative effects in the long run and during the transitional period. In this respect, our theoretical model supports the findings of the empirical literature, which were inconclusive in nature. In the special case where the observations are made from LDCs, it is theoretically correct to expect that defense spending may have a negative effect on the convergence process.
References


