Working Papers in Economics

A Romerian Contribution to the Empirics of Economic Growth

Bahar Bayraktar Sağlam, Hacettepe University
I. Hakan Yetkiner, Izmir University of Economics

Working Paper # 12/01

January 2012

Izmir University of Economics
Department of Economics
Sakarya Cad. No: 156
35330 Balçova Izmir
Turkey

Working Paper # 12/01

A Romerian Contribution to the Empirics of Economic Growth

Abstract

Mankiw Romer and Weil (1992) made the Solovian set up widely-used to test the determinants of economic growth and the speed of convergence. Subsequently, in almost all convergence studies, an exogenously growing technology is assumed and this component is treated as part of the constant term. In this study, we expand the Mankiw Romer and Weil (1992) set-up through a Solovianized Romer (1990) framework, which allows us to decompose the exogenously growing technological progress. Within this framework, the growth rate of technology depends on the characteristics of the R&D sector, including the share of labor devoted to R&D activities. We estimate the convergence equation derived from Solovianized Romer model for 31 OECD countries for the period 1980-2008 by applying the system GMM approach. The empirical findings of the model supports the conditional convergence hypothesis, but predicts a much lower convergence rate (0.01) than that predicted by the existing empirical growth literature (0.02). The model supports the positive and significant role of R&D on economic growth. Another contribution of the model is the elegant introduction of human capital to the convergence equation.

Key Words: Convergence, Economic Growth, Exogenous Technological Change.
JEL Classification: O30, O47, O50, C23.

Bahar Bayraktar Sağlam
Department of Economics
Hacettepe University
06800, Ankara, Turkey
Email: sbahar@hacettepe.edu.tr

I. Hakan Yetkiner
Department of Economics
İzmir University of Economics
Izmir, Turkey 35330
Email: hakan.yetkiner@ieu.edu.tr

1 Introduction

Empirical studies on economic growth have expanded rapidly since the early 1990s. In 1990, only 1 publication appeared in SSCI- indexed economics journals with key words “empirical” and “economic growth” in topic search. By 1995, this had risen to 49. After this, the number increased over successive five year periods to 71, 131, and 225 and, by 29 December 2011 a total of 2223 articles in this area had been published in SSCI-indexed economics journals. The quick development of the new economic growth theory, the availability of richer databases, e.g., Summers and Heston (1988, 1991), and improvements in econometric techniques, which provide a higher degree of precision and greater confidence in the analysis findings, have contributed to the rapid expansion of the empirical studies in that direction.

The neoclassical growth theory (i.e., Solow framework), under the assumptions of constant returns to scale and diminishing marginal returns to capital, reveals that the saving rate (=investment rate) is the major determinant of economic growth in the transitional period, and that technological progress, which is regarded exogenous to the system, is the determinant of the long run growth.[1] Two (interrelated) empirical research strands have emerged from this theory. The first strand of empirical studies aimed to determine the sources of international differences in income per capita. Even though the findings were not fully conclusive, a wide range of studies have demonstrated that investment in physical and human capital, innovation and R&D, macroeconomic policies (such as inflation, fiscal policy and budget deficits), trade openness, the institutional framework, geography, demographic trends, political and socio-cultural factors are all likely to have important impacts on the process of economic growth.[2]

The second strand investigated whether low-income economies grow faster than the high-income ones, as the neoclassical growth theory conjectures that countries having similar characteristics but lower initial physical capital will grow at a higher rate due to diminishing marginal returns premise. This argument has quickly become dubbed ‘Convergence Analysis’. Early works in

[1] On the other hand, the endogenous growth theory, triggered by the work of Romer (1986), mainly through introducing non-diminishing returns to one or more factors of production, has proposed several other mechanisms of endogenous growth, such as human capital (Lucas, 1988), knowledge capital (Romer, 1990), quality ladders (Aghion and Howitt, 1992), and public infrastructure (Barro, 1990).
[2] For a survey on the determinants of economic growth, see Sala-i-Martin (1997), Durlauf *et al*. (2005) and Petrakos *et al*. (2007).

this direction were Baumol (1986), Abramovitz (1986) and de Long (1988). An enormous body of research started, however, after Barro and Sala-i-Martin (1992) and especially Mankiw, Romer and Weil (1992).[3] It was MRW (1992) that rapidly emerged as the fundamental empirical framework to test for convergence, as their suggested equation was fitting very well for empirical purposes, e.g., data availability.

MRW (1992) estimates an augmented Solow model, which includes human capital stock in addition to physical capital stock. Their model reveals that convergence in income per capita is determined by population growth, accumulation of physical capital and of human capital. Within this set up, they find strong evidence for (conditional) convergence: countries with similar technologies and rates of physical and human capital accumulation converge in income per capita. Given the Solovian set up, and the fact that technology is exogenous, it was simplifying for MRW (1992) to assume that technological growth does not vary across countries (in particular, it was taken as 0.02).[4] Subsequently, following MRW (1992), most empirical studies, concerned with the convergence issue or on sources of international differences in income per capita, not only treated technology as an exogenous component, but in addition, handled it as only a component in the constant term in econometric sense. Islam (1995) and Caselli *et al*. (1996) improved MRW (1992) through using a dynamic panel data technique for convergence analysis by allowing for individual country effects to capture the technology differences across countries. Their findings yield higher rates of conditional convergence since panel data contribute to the correction of the omitted variable bias that exists in the cross-section regressions.

Many convergence studies, such as Islam (1995), Caselli *et al*. (1996), Murthy and Chien (1997), Nonneman and Vanhoudt (1996), and Keller and Poutvaara (2005) estimated the convergence rate in the range 0.02 to 0.10. These studies used equations in which convergence takes place through the adjustment of the capital output ratio rather than changes in technology or its determinants.[5] We believe, as a result of this stance, a large body of empirical studies on

---

[3] Henceforth, we will use MRW (1992) instead of Mankiw, Romer and Weil (1992).

[4] Clearly, there are many TFP studies showing country-specific TFP growth, such as Fagerberg (1994), Young (1994) and Young (1995).

[5] Bernard and Jones (1996) demonstrated that growth rates of output among OECD countries converge, while the growth rates of manufacturing technologies exhibit markedly different time profiles. McQuinn and Whelan (2007) estimated the rate of conditional convergence based on the dynamics of capital output ratio. However, they defined the output per worker as a function of the level of technological efficiency and of the capital output ratio, previously used by Hall and Jones (1997). They found higher speed of conditional convergence than that of the Solow model and the previous studies.

conditional convergence has overemphasized the role of capital accumulation, while underestimating the role of technological change.

This study is not alone in recognizing the weakness of employing a constant and non-varying rate of technological change across countries in convergence studies, and thus opposing its adoption.[6] One good example is Bloom *et al*. (2002), which objected to both the idea of *identical* rate of technological progress in every country, and the *fixed effects approach* adopted by panel data studies such as Islam (1995) and Caselli *et al*. (1996), which allow for TFP differentials across countries on the one hand, but assume that these differentials persist indefinitely, on the other. Bloom *et al*. (2002) shows that technological diffusion implies a form of conditional convergence as lagging countries catch up with technological leaders and they find strong evidence of technological diffusion but not full convergence.[7]

Another study which strongly rejected the assumption of common exogenous technology for all countries is Dowrick and Rogers (2002). This study aimed to address the contribution of technological catch up to conditional convergence, arguing that differences in the growth rates of technology among economies may result from technological catch up. In particular, they argued that the growth rate of technology depends on the technology gap between the leader and the follower. In this respect, Dowrick and Rogers (2002) not only consider the country-specific part of the technological growth, but also model the technological gap as an important determinant of technology growth. The MRW (1992) framework was re-estimated using this modification, and a higher convergence rate was arrived at.[8]

We argue that one major reason for the convergence literature persistently assuming constant and identical growth rate is the over simplicity of the Solovian framework, which does not allow for the differentiation of technological change across economies under the exogenous technological change assumption. In that respect, we argue that the literature needs an augmented framework that allows the differentiation of economies on the

[6] See, for example, Howitt (2000), documenting that technology differences are significant across countries.

[7] We argue that Bloom *et al*. (2002) have two weaknesses. First, it uses the total factor productivity of production function for deriving technological differences. The study assumes that differences in total productivity between countries reflect differences in technology, which is a very broad interpretation of total factor productivity. Second, Bloom *et al*. (2002) is not based on a general equilibrium modeling.

[8] We argue that Dowrick and Rogers (2002) only consider the technological gap between the leader and the follower, but do not consider any other variable that may contribute to

sources of economic growth in general, and on technological change in particular. On the theoretical side, recall that the emergence of first-generation endogenous growth models has emphasized R&D activities as major force behind economic growth (cf., Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)).[9,10] Since the scale-effect prediction of those models were not supported by the data (cf., Jones (1995)), this led to the emergence of second generation endogenous growth or semi-endogenous growth models, which assumes that the rate of technological progress in any individual country depends on its research intensity, that is, the proportion of the labor force employed in the R&D sector and the proportion of income devoted to R&D sector.[11] For example, Ulku (2007) notes that two effective measures of R&D intensity are the share of researchers in the labor force, and the share of R&D expenditures in GDP. This paper indeed uses these two variables for explaining convergence in empirical part of the study.

Another relevant group of studies concern the empirical application of semi-endogenous growth models. While Jones (2002), Kim (2008) and Kim (2011) all focus on the role of R&D intensity on economic growth under the umbrella of the semi-endogenous growth set-up, they do not adopt a general equilibrium framework.[12]. By adopting growth accounting approach, Jones (2002) demonstrated that R&D intensity and educational attainment explain 80% of the US economic growth. Similarly, within the area of growth accounting, Kim (2008) finds that 80% of the economic growth in Korea can be attributed to the transitional dynamics. Kim (2011) used the semi endogenous growth model of Jones (2002) for both time series and panel data regressions to test the model for US, Japan, Canada, France and UK for the period 1948-2006. His results suggest higher rate of convergence compared to the MRW (1992), and also that R&D expenditure has a significant role in explaining economic growth. These studies also inspire us in the empirical part of the work.

---

technological change. Perhaps that is why they obtained (surprisingly) higher convergence rate than that which the literature actually suggests.

[9] We are aware of early endogenous growth literature of the 1960s, such as Arrow (1962). We skip to discuss them here as they never succeed to show rigorously endogenous growth, though they had the idea. See Schneider and Ziesemer (1995) for a survey of this literature.

[10] Gong *et al*. (2004) modified the Romer endogenous growth model in order to estimate the preference and technology parameters of the Romer (1990) model for the US and Germany data for the time period January 1962 to April 1996. They concluded that the stock of human capital in R&D sector (so-called the scale effect) is not significant and that the stock of human capital devoted to the generation of knowledge must increase to sustain economic growth. Yet, the study is improper for using in convergence analysis by construction, as it relies on R&D-based endogenous growth model of Romer.

[11] For example, Ha and Howitt (2007), Howitt (2000), Zachariadis (2004), Madsen (2008) and Ulku (2007).

[12] This present presents an equilibrium solution of Jones (2002) in Annex C.

This paper aims to overcome the aforementioned weaknesses of convergence analysis. In particular, we aim to study the contribution of technology to convergence and economic growth under exogenous technological change. To this end, we first develop a framework, which can easily be tested. In the theoretical part of the paper, we incorporate the two strands of theoretical growth literature, namely, the neoclassical growth model of Solow (1956) and the endogenous technological change model of Romer (1990). The exogenous allocation of consumption-saving tradeoff, borrowed from the first strand, is incorporated to the second strand, which we suggest to call Solovianized Romer model. Our aim in doing that was to develop a convergence equation that fits well with what has been done since MRW (1992).

Through a Solovianized Romer framework, this study indeed aims to overcome the two aforementioned weaknesses of the convergence literature. First, it develops a theory-backed convergence equation in order to allow the rate of technological change to vary across economies, depending on the characteristics of R&D sector in particular.[13] Second, this study introduces human capital in a more elegant way in the convergence equation. In particular, we define human capital à la Romer (1990), which allows its extension in several respects. We consider the treatment human capital as a simple duplicate of physical capital, à la MRW (1992), as over-mechanical and undervaluing the different role played by human capital. In this respect, our model is a clear improvement over MRW (1992).

The paper presents no less than four different versions of the Solovianized Romer model. In the main body, we present the basic theoretical framework, which to a large extent reproduces the convergence equation of MRW (1992) with a theoretical support to treat technological progress asymmetric across economies. We defer the presentation of variations in the basic framework, which are not tested in this paper, to the appendices. We are aware of the fact that the basic model may be considered as the most unrealistic one among all varieties. Nonetheless, our aim in this paper is not solely to develop the best empirical model but to develop the empirical backbone of the theoretical part. The basic model derives the MRW-like convergence equation at the cost of assuming exogenous allocation of human capital between final good and R&D sectors. Annex A extends the basic model by including unskilled labor as well as human capital. The basic model produces a convergence equation that yields unitary coefficients for some variables. This exercise proves that the unit

[13] There are many studies, such as Nonneman and Vanhoudt (1996), Murthy and Chien (1997) and Keller and Poutvaara (2005), in which the stock of knowledge (or its change) is introduced to the model directly (and exogenously) through the production function. In contrast, in this study, the know-how is introduced to the model through a dynamic general equilibrium model.

coefficient should not be taken literally. Annex B presents the model with endogenous allocation of human capital between these two sectors (which is perhaps the genuine Solovian Romer model). In that extension, we are able to derive an equation for long determinants of economic growth; however, obtaining a convergence equation in the usual form is not possible. In fact, we had no such aim in this work, as endogenous allocation of human capital implies endogenous technological change, which is not a truly prominent framework for empirical studies. Finally, Annex C provides the competitive equilibrium solution of Jones (2002) under exogenous allocation of human capital between final-good and R&D sectors. This exercise shows that, if required, the so-called scale-effect can easily be removed from the long-run determinants of growth or convergence equations, although we relied on the basic model in the empirical part of this paper. At the bottom line, Annex A and Annex C do show that extension of the basic setup does not qualitatively change the equation derived in the basic setup.

The basic empirical convergence equation necessarily implies that technological change is different across economies, which, to our knowledge, has been rarely mentioned in the literature. We believe that this is an important step towards understanding the genuine convergence mechanism, as developing countries provide a minimal contribution to technological progress, and these economies have a low R&D intensity, by and large. Our framework avoids the need for the assumption of the constant technology parameter in MRW-like convergence equation, since the solution of the model reveals that technological progress depends on the characteristics of the R&D sector in general, and the proportion of R&D personnel in the labor force in particular. This study addresses the apparent lack of rigorous econometric work aimed at assessing the impact of different technological intensities on economic growth and convergence. More than this, the point in the theoretical framework at which Solow (1956) meets Romer (1990) brings into forefront the role of human capital (in the final-good and R&D sectors) in convergence in a more elegant way, since human capital is no longer treated solely as a duplicate of physical capital, as has been the case since MRW (1990)).

The empirical part of the paper estimates the Solovianized Romer model for 31 OECD countries across the period 1980-2008 by employing system GMM approach. We have run the basic theoretical model in three different versions. The dependent variable is the log differences in GDP per working person for 3 versions. In Model 1, the dependent variable is regressed on the logarithm of the initial GDP, the investment to GDP ratio, the share of final good workers in the labor force and the share of R&D workers in the labor force. In Model 2, the dependent variable is regressed on the logarithm of the initial GDP, the investment to GDP ratio, the human capital (proxied by the secondary enrollment rate) and the share of R&D workers in the labor force. In Model 3, the dependent variable is regressed on the logarithm of the initial GDP, the investment to GDP ratio, the human capital (proxied by the secondary enrollment rate) and the share of R&D expenditure in GDP. Our findings can be summarized as follows:

1. All runs imply a convergence rate lower than that which is suggested by the literature in general.
2. The investment rate has a positive and statistically significant contribution to convergence in all runs.
3. R&D has a positive and statistically significant impact on convergence in all runs regardless of the proxy used. Both the shares of labor and of income devoted to R&D have a positive and significant role on growth.
4. The role of human capital on convergence is positive but statistically insignificant in Model 1 and Model 2, but positive and statistically significant in Model 3.

Our results suggest a deviation from findings in the existing literature. Nonetheless, we are aware of the fact that our results can only be considered preliminary, and that a more thorough test of the framework is needed. Our initial findings suggest, however, that the framework has the potential to yield results that are more realistic in terms of convergence behavior. In this respect, this paper should only be considered as a first step in the process of developing a more realistic test of convergence hypothesis. The organization of the paper is as follows. Section 2 presents a theoretical framework, which develops a growth equation and a convergence equation for empirical use. Section 3 is reserved for the empirical research. We show that the convergence rate is smaller than that which is suggested by the conventional literature. Section 4 concludes the paper.

2 The Basic Model

As the Romer (1990) model is widely-known we will be as compact as possible in its presentation. Following Romer (1990), we assume that the production technology has an additively separable characteristic:

$$Y = H_Y^{1-\alpha} \sum_{i=1}^{A(t)} X_i^{\alpha} \qquad 0 < \alpha < 1 \qquad (1)$$

where Y is final good (GDP), H_Y is the amount of human capital used in final good production, $1-\alpha$ is the respective production elasticity of that human capital, X_i are intermediate goods (varieties), and $A(t)$ is the number of intermediate goods at time t.[14] It should be noted that the form of (1) is Cobb-Douglas, $Y = H_Y^{1-\alpha}\left(X^e\right)^{\alpha}$, for $X^e = \left(\sum_{i=1}^{A(t)} X_i^{\alpha}\right)^{1/\alpha}$, where X^e may be called efficient capital stock. We assume that human capital is allocated between the final-good and R&D sectors, that is,

$$H_Y = \theta_Y \cdot \overline{H} \text{ and } H_{R\&D} = \theta_{R\&D} \cdot \overline{H} \qquad (2)$$

where H_Y ($H_{R\&D}$) is amount of human capital employed by the final-good (R&D) sector, θ_Y ($\theta_{R\&D}$) is the respective share of human capital, and $\overline{H}$ is the stock of human capital, which is constant. Note that $\theta_Y + \theta_{R\&D} = 1$ at all times. In original Romer model, the allocation of human capital between final good sector and R&D sector is endogenous. Below, given our aim to derive an empirically usable convergence equation, we will assume that the tradeoff is exogenous to the model.[15] Hence, θ_Y and $\theta_{R\&D}$ are constant.

Final Good Sector

We assume that there is perfect competition in the final-good sector and we take final output to be the numéraire. Hence, the profit equation is

$$\Pi_Y = \overline{H} \cdot \left[\theta_Y^{1-\alpha} \sum_{i=1}^{A(t)} x_i^{\alpha} - w_Y \cdot \theta_Y - \sum_{i=1}^{A(t)} p_i \cdot x_i\right] \qquad (3)$$

where x_i are intermediate goods in per capita, w_Y is the real wage rate for the skilled labor in final good sector, and p_i is the user cost of intermediate-good i. The level of demand for each intermediate and human capital (employed at final-good production) follows directly from the first order profit maximization conditions:

[14] We purposefully refrained from defining unskilled labor as the third argument in the production function, à la Romer (1990), to keep derivations simple. This simplification will result in the drawback of having a convergence equation in which some coefficients are unitary. We show in Annex A that unit values of these coefficients should not be taken literally and that it is due to our simplified approach.

[15] Interested readers may refer to Annex B to see how results do change when labor allocation is endogenous.

$$\frac{\partial \Pi_Y}{\partial H_Y} = (1-\alpha)\cdot \theta_Y^{-\alpha} \cdot \sum_{i=1}^{A(t)} x_i^{\alpha} - w_Y = 0 \qquad (4a)$$

$$\frac{\partial \Pi_Y}{\partial X_i} = \alpha \cdot \theta_Y^{1-\alpha} \cdot x_i^{\alpha-1} - p_i = 0 \qquad \forall i \qquad (4b)$$

The above-given first-order profit maximization with respect to H_Y and X_i are inverse demand functions for human capital (employed at final-good production) and any individual intermediate i.

Intermediate-good Sector

We assume that the intermediate-good producing sectors only use 'raw' capital in order to produce an intermediate good: $K_i = X_i$ (or $k_i = x_i$ in per capita terms), where K_i measures the total amount of raw capital going into intermediate good of type i. Raw capital can be rented at the real rate of interest r plus depreciation δ: $r' \equiv r + \delta$. Hence, r' is the rental rate of capital. We assume that each intermediate-good producer has monopoly power right over the production and sale of the good X_i, as the patent (the blueprint) of the product belongs to it. Hence, the seller of the intermediate good faces a downward-sloping demand curve (cf., equation (4b)). Therefore, the profit that the i^{th} monopolist has to maximize is $\Pi_i = \overline{H} \cdot [p_i \cdot x_i - r' \cdot x_i]$. The profit maximizing price for the intermediate good i is obtained as $p_i = p = \frac{r'}{\alpha}$, which underlines the fact that price is identical across intermediates. Note that substitution of price information in equation (4b) reveals that $x_i = x = \theta_Y \left(\frac{\alpha^2}{r'}\right)^{\frac{1}{1-\alpha}}$, the quantity of each intermediate are identical across varieties. Identical prices and quantities across intermediates naturally imply that profits are identical across intermediates as well, $\pi_i = \pi = \frac{\Pi_i}{\overline{H}} = (1-\alpha)\cdot\alpha\cdot\theta_Y^{1-\alpha}\cdot x^{\alpha}$. Given our findings that price, quantity and profit are identical across intermediates, it must be true that $K = \sum_{i=1}^{A(t)} K_i = \sum_{i=1}^{A(t)} X_i = X \cdot A$ or $k = \sum_{i=1}^{A(t)} k_i = \sum_{i=1}^{A(t)} x_i = x \cdot A$. Using this information in (1) yields

$$y = \theta_Y^{1-\alpha} \cdot x^{\alpha} \cdot A \text{ or } y = \theta_Y^{1-\alpha} \cdot k^{\alpha} \cdot A^{1-\alpha} \qquad (5)$$

Note from above derivations that $x = x(r)$ for a given θ_Y. We will below show that r is constant at steady state. Hence, y and k grow at the rate of A at steady state.

R&D Sector

À la Romer (1990), we will define the knowledge production function as $\dot{A} = \eta \cdot H_{R\&D} \cdot A$. Given (2), the growth rate of knowledge accumulation, $\hat{A} = \eta \cdot \theta_{R\&D} \cdot \overline{H} \equiv g$, is exogenous, as in MRW (1992).[16] The difference is that we know the components of it (see Annex B for endogenous allocation of human capital and technological progress).

Given the standard perfectly competitive R&D sector assumption and the knowledge production function, the value of the i[th] patent may easily be determined as $V_{R\&D,i}(t) = \dfrac{\overline{H} \cdot \pi(t)}{r(t)}$, which is an arbitrage rule stating that the return out of investing an amount equal to the value of patent in the 'financial market' at time t, $r(t) \cdot V_{R\&D,i}(t)$, must be equal to profit $L \cdot \pi(t)$ derived from that patent at time t. The arbitrage rule is valid as long as per capita profit is constant. Recall that $\pi = \pi(r, \theta_Y)$. Hence, as long as the real rate of interest r is constant, which is true at steady state, the arbitrage condition would be as derived above. Finally, note that using the arbitrage rule in the first-order profit maximization condition in R&D sector, $V_{R\&D} \cdot \eta \cdot A = w_{R\&D}$, implies $\dfrac{\overline{H} \cdot \pi(t)}{r(t)} \cdot \eta \cdot A = w_{R\&D}$, where $w_{R\&D}$ is the real wage rate in the R&D sector.

Consumption-saving tradeoff

We assume that the consumption-saving tradeoff is exogenous, á la Solow (1956). This assumption allows us to simplify Romer (1990) considerably, without losing the main deriving forces of the role of technological progress on economic growth. We assume that capital accumulation is led by:

$$\dot{K} = s \cdot Y - \delta \cdot K \qquad (6)$$

where K is capital, s is the exogenous saving (investment) rate, Y is output and δ is depreciation rate of capital.

---

[16] A hat on top of a variable indicates the growth rate.

Long-run Equilibrium

Recall that knowledge growth rate is $\hat{A} = \eta \cdot \theta_{R\&D} \cdot \overline{H}$. Let us now determine the steady state value of unknowns of the model. To this end, first, by using the capital accumulation function in (6), we can show that

$$\hat{\tilde{k}} = s \cdot \theta_Y^{1-\alpha} \cdot \tilde{k}^{\alpha-1} - (\delta + g) \quad (7)$$

where a tilde on top of a variable defines per efficient capita, e.g., $\tilde{k} = \frac{K}{A \cdot \overline{H}}$. It is well-known that capital per efficient capita does not grow at the steady state. Hence, the steady state value of capital per efficient capita is $\tilde{k}_{ss} = \theta_Y \cdot \left(\frac{s}{\delta + g}\right)^{\frac{1}{1-\alpha}}$. By construction, this may be interpreted as the supply of capital per efficient capita in the model. We need to define also demand for capital. Recall that we had already indicated that $k = \sum_{i=1}^{A(t)} k_i = \sum_{i=1}^{A(t)} x_i = x \cdot A$, which implies $\tilde{k} = x$. Hence, at steady state (capital accumulation still has transitional dynamics), $\left(\frac{s}{\delta + g}\right)^{\frac{1}{1-\alpha}} \cdot \theta_Y = \theta_Y \cdot \left(\frac{\alpha^2}{r'_{ss}}\right)^{\frac{1}{1-\alpha}} \Rightarrow \quad r'_{ss} = \frac{\alpha^2(\delta + g)}{s}$. Note that r'_{ss} responds negatively to an increase in saving rate and positively to the exogenous growth rate. The rest of the model can be solved through substitution. In particular, one should be able to get $\tilde{y}_{ss} = \theta_Y \cdot \left(\frac{s}{\delta + g}\right)^{\frac{\alpha}{1-\alpha}}$. Finally, it is straightforward to show that $\hat{y}_{ss} = \hat{k}_{ss} = \hat{A} \equiv g$. The growth rate of GDP increases if (i) the productivity of R&D sector and (ii) the size or the share of human capital increases. It can be clearly shown that the real rate of interest, the price of intermediate-good, the quantity of each intermediate-good and the profit for each intermediate are all constant at the steady state.

2.1 Determinants of Steady State Growth

Recall that we found the steady state value of output per efficient capita as $\tilde{y}_{ss} = \theta_Y \cdot \left(\frac{s}{\delta + g}\right)^{\frac{\alpha}{1-\alpha}}$. By taking its natural log, and after a few algebraic transformations, we can show that

$$\ln y_{ss} = a + g \cdot t + \ln[\theta_Y] + \frac{\alpha}{1-\alpha}\ln[s] - \frac{\alpha}{1-\alpha}\ln[\delta + g] + \varepsilon \qquad (8)$$

where $Ln[A(0)] = a + \varepsilon$ and $g = \eta \cdot \theta_{R\&D} \cdot \overline{H}$. Equation (8) may be considered as a base for studying long-run determinants of economic growth, á la MRW (1992).[17] The main contribution of equation (8) is that the exogenous growth rate is decomposed into its components. In addition, there are two additional added values in (8). First, human capital is incorporated into the model through a general equilibrium modeling approach. Second, the three-sector structure of Romer framework has obvious advantages over the one-sector Solow framework in terms of flexibility for extending the framework in several research directions. All in all, we believe (8) is richer than the Solow version.

2.2 Convergence

What is additionally assumed in (8) is the contribution of components of technological progress to long run economic growth. However, technological progress is also important in the transition to steady state. In particular, it is interesting to consider the major factors that contribute any significant role in convergence of economies. As convergence derivations are widely known, we will keep (descriptions of) these to a minimum. Recall that capital accumulation function is given by (7). We first need to express (7) in terms of $\tilde{y}$, as it is better to directly work with GDP per efficient capita for convergence analysis (in the final step, we will transform it into GDP per capita, which is what practically measurable). To this end, through using log differentiated production function, that is, $\hat{\tilde{y}} = \alpha \cdot \hat{\tilde{k}}$ (recall that $\hat{\tilde{y}} = \dot{\tilde{y}} / \tilde{y}$ and $\hat{\tilde{k}} = \dot{\tilde{k}} / \tilde{k}$), and after simple arithmetic operations, we may re-express (7) in terms of $\tilde{y}$:

$$\frac{dLn(\tilde{y})}{dt} = \alpha\left[s \cdot \theta_Y^{\frac{1-\alpha}{\alpha}} \cdot e^{\frac{\alpha-1}{\alpha}Ln(\tilde{y})} - (\delta + g) \right] \equiv \phi(Ln(\tilde{y})) \qquad (9)$$

[17] Recall that in the standard Solow set-up, the steady state value of output per efficient capita is obtained as $\tilde{y}_{ss} = \left(\frac{s}{\delta + g}\right)^{\frac{\alpha}{1-\alpha}}$ for $Y = K^{\alpha}(A \cdot \overline{H})^{1-\alpha}$ and $A = A(0) \cdot e^{g \cdot t}$. Hence, the determinants of economic growth are found as $\ln y_{ss} = a + g \cdot t + \frac{\alpha}{1-\alpha}\ln[s] - \frac{\alpha}{1-\alpha}\ln[\delta + g] + \varepsilon$, where $Ln[A(0)] = a + \varepsilon$. Note that the determinants of g and $A(0)$ are undefined in that case.

The differential equation in (9) is not linear. Through log-linearization, we find that

$$\frac{dLn(\tilde{y})}{dt} \approx -(1-\alpha)(\delta+g)\left[Ln(\tilde{y}) - Ln(\tilde{y}_{ss})\right] \qquad (10)$$

Let us now define $\nu = (1-\alpha)(\delta+g)$. Just to understand the full implications of (10), suppose that $Ln(\tilde{y}) < Ln(\tilde{y}_{ss})$. As $Ln(\tilde{y}) - Ln(\tilde{y}_{ss})$ is negative, $-\nu \cdot \left[Ln(\tilde{y}) - Ln(\tilde{y}_{ss})\right]$ would be positive. More than this, the higher the difference between $Ln(\tilde{y})$ and $Ln(\tilde{y}_{ss})$, the higher would be $-\nu \cdot \left[Ln(\tilde{y}) - Ln(\tilde{y}_{ss})\right]$. Then, an economy further away from its steady state would have a higher growth rate. That is, the speed that an economy converges to its own steady state is $cr = \frac{d(\hat{\tilde{y}})}{dLn(\tilde{y})} \approx -\nu$. The solution of the linearized differential equation in (10) yields

$$Ln\left(\frac{Y(t)}{\overline{H}}\right) - Ln\left(\frac{Y(0)}{\overline{H}}\right) = \beta_0 + \beta_1 \cdot Ln\left(\frac{Y(0)}{\overline{H}}\right) + \beta_2 \cdot Ln(\theta_Y) + \beta_3 \cdot Ln(s) + \beta_4 \cdot Ln(\delta+g) \qquad (11)$$

where $\beta_0 = g \cdot t + \left(1-e^{-\nu \cdot t}\right)Ln(A(0))$, $\beta_1 = -\left(1-e^{-\nu \cdot t}\right)$, $\beta_2 = -\beta_1$, $\beta_3 = -\left(1-e^{-\nu \cdot t}\right)\left(\frac{\alpha}{1-\alpha}\right)$ and $\beta_4 = -\beta_3$. The LHS of (11) is the growth rate of per capita income relative to initial level. Determinants of this change in level of per capita take place on the RHS. There are two constants in β_0: total growth rate between the initial time and ending time, $g \cdot t$, and $\left(1-e^{-\nu \cdot t}\right) \cdot Ln(A(0))$. β_1 is the coefficient of initial level of income per capita. Notably, this coefficient is negative, which is consistent with the convergence idea. Coefficient β_2 indicates the contribution of human capital on convergence. The higher the share of human capital used in final good production, the higher is the growth rate. Finally, coefficient β_3 and β_4 show the contribution of investment rate and effective depreciation on convergence, respectively. There is at least two-fold improvement relative to MRW (1992). First, g has been decomposed into its components (see equation (12) below for a more concrete illustration of the convergence equation used for empirical purposes). Contrary to many previous studies, we do not have to consider this the same and constant across economies, which was previously the common assumption. Second, we are able to incorporate human capital into the model in a more elegant way in the convergence equation. Another value added over the existing literature is that the three-sector nature of the model allows further extensions in several directions by using the competitive equilibrium approach.

3 Data, Methodology and Findings

In this section, we estimate equation (11), placing special emphasis on the R&D intensity of the countries. In these estimations, we no longer assume constant and identical technology growth, g, across economies. In particular, any particular economy is likely to have a different rate of technological change at any one time due to variation in its research intensity. We recalculate the rate of convergence if there is no constant technological progress.

We estimate equation (11) for 31 OECD countries over the period 1980-2008.[18] According to standard practice in the empirical growth literature, the data is transformed into five-year averages over the period 1980-2008 in order to eliminate the cyclical component. The definition of the variables is presented in Table 1 and Table 2 is reserved for the sample statistics.

Table 1 Definition of Variables

Variable	**Definition**	**Data Source**
$Ln(y)$	Logarithm of growth in real GDP per head of population aged 15-64 years expressed in 2000 purchasing power parities	OECD Annual National Accounts
$Ln(y_{t-1})$	logarithm of lagged growth in real GDP per head of population aged 15-64 years expressed in 2000 purchasing power parities	OECD Annual National Accounts
$Ln(s)$	Gross fixed investment share of GDP	World Development Indicators Database
$Ln(h_1)$	Secondary school enrollment rate	Barro-Lee Education Dataset (2010)
$Ln(h_2)$	The share of final good workers in the labor force.	Own calculations where $\theta_Y + \theta_{R\&D} = 1$
$Ln(R\&D_1)$	The share of R&D in the labor force	OECD Main Science and Technology Indicators database
$Ln(R\&D_2)$	The share of R&D expenditure on GDP	OECD Main Science and Technology Indicators database

[18] The data set is a slightly unbalanced panel with 186 observations where the data is missing for some countries for some periods. The list of countries is in Annex D.

Table 2 Basic Statistics				
Variables	Mean	Standard Deviation	Min	Max
GDP	21291	8858	5326	62731
INV	22.8	3.8	37	17
h_1	45.6	13.9	8.2	88
h_2	995.0	2.68	985.0	999.5
$R\,\&\,D_1$	4.9	2.68	0.44	15
$R\,\&\,D_2$	1.56	0.83	0.2	3.9

The model to be estimated has the following form:[19]

$$\ln y_{it} - \ln y_{i0} = \beta_0 + \beta_1 \ln y_{i0} + \beta_2 \ln s + \beta_3 \ln h + \beta_4 \ln[\delta + r\,\&\,d] + \mu_i + \phi_t + \varepsilon_{it} \qquad (12)$$

where μ_i and ϕ_t represent country specific and time specific effects, and where $\beta_0,...\beta_4$ are parameters to be estimated. To estimate the parameters of the above equation, we adopt the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). The system GMM has been selected as, firstly, it provides consistent and efficient parameter estimates, even in the presence of measurement error and endogenous regressors. Second, it is highly recommended for empirical growth models (Bond *et al*., 2001). Third, it particularly suits the short time dimension panel data sets. A final benefit of this system is its greater efficiency in dynamic panel data, compared to the difference GMM, as lagged levels in the latter can be weak instruments for subsequent changes (Blundell and Bond (1998, 2000), Blundell *et al*. (2000)).

System GMM procedure consists of a joint estimation of the equation in first-differences and in levels. For the equations in first-differences, the lagged levels of the regressors and for the equations in levels, the lagged first-differences of the explanatory variables are used as instruments. The consistency of the system GMM estimator depends on the two conditions. First, there should be no serial correlation in the error term. Second, the instruments should not be correlated with the error term. There are two key diagnostics to check for these problems. The Arellano-Bond test for serial correlations examines the first and second order correlations of the first differenced residuals. The correct specification of instruments and the validity of the instruments are checked by the conventional Hansen test of over-

[19] As in the MRW (1992) the depreciation rate is assumed to be 0.03.

identifying restrictions. In addition, the number of cross section units should be larger than the number of instruments.[20] Table 3 below presents system GMM results[21]. The dependent variable is the log differences in GDP per working person for 3 versions. In Model 1, the dependent variable is regressed on the logarithm of the initial GDP, the investment to GDP ratio, the share of final good workers in the labor force and the share of R&D workers in the labor force. In Model 2, the dependent variable is regressed on the logarithm of the initial GDP, the investment to GDP ratio, the human capital (proxied by the secondary enrollment rate) and the share of R&D workers in the labor force. In Model 3, the dependent variable is regressed on the logarithm of the initial GDP, the investment to GDP ratio, the human capital (proxied by the secondary enrollment rate) and the share of R&D expenditure in GDP.

Table 3 System GMM Estimation of the Solovenized Romer Model

	Dependent Variable: log differences in GDP per working person 1980-2008		
	Model 1	**Model 2**	**Model 3**
$Ln(y_{t-1})$	-0.2431*** (0.0896)	-0.2231*** (0.0758)	-0.1499*** (0.0425)
$Ln(s)$	0.5013** (0.2167)	0.6648*** (0.2324)	0.3841** (0.1431)
$Ln(h_1)$		0.0150 (0.0914)	0.0931** (0.0494)
$Ln(h_2)$	0.1607 (0.1916)		
$Ln(R\&D_1 + \delta)$	0.1332*** (0.0442)	0.1523** (0.0655)	
$Ln(R\&D_2 + \delta)$			0.1079** (0.0522)
Implied ν	0.01	0.01	0.01
Number of Observations	155	155	155
Number of Groups	31	31	31
Number of Instruments	13	13	13
Hansen test p value	0.39	0.37	0.50
Difference Hansen p value	0.52	0.62	0.54
M2	0.271	0.319	0.887

Note: Heteroskedasticity-consistent standard errors are in parentheses. The test statistics for second order correlation is given by M2 and p values in brackets. *** and ** indicate that the coefficient is significant at 1 and 5 percentage, respectively. We report the results using 2nd and 3rd lags of the variables.

[20] We have use the command of collapse available in Stata (v.10) as mentioned in Roodman (2009).

[21] Roodman (2006) "xtabond2" command was used in Stata (v.10) for the system GMM estimations. Windmeijer (2005) is implemented for the small sample correction.

Table 3 shows that the coefficients on initial income have the expected negative sign and are highly significant in all models, evidence of conditional convergence. The estimation of all models yields a convergence rate of 1% per year, a figure which is significantly lower than the 2% per year generally found in the literature. In other words, replacing the constant technology growth assumption of MRW (1992) with the findings of the Solovianized Romer model, where the technological growth depends on the share of income (labor) devoted to R&D, reveals a lower rate of convergence.

To check for the consistency of the results, we also replicate the basic MRW (1992) model with human capital accumulation by using the data set used in this paper. Table 4 compares the findings for the convergence rate under the period 1980-2008 for OECD countries. The estimation of the model, under the assumption of exogenous growth rate of technology, finds a convergence rate to be 0.02. But, once the $r\&d$ intensity is substituted for the growth rate of technology, the estimation of the model reveals a lower convergence rate, namely, 0.01.

Table 4. Convergence Rate under MRW versus Our Model

	g (the growth rate of technology)	
	0.02	$r\&d$ **intensity**
Implied v	0.02	0.01

Our empirical estimations point out that the investment rate has a significant and positive effect on the growth rate of GDP per capita in all runs. The estimated coefficient for the physical capital investment rate is positive and significant, and, in addition it is greater than that of the coefficients of the human capital and the share of R&D personnel in the labor force.

On the other hand, in Model 1, the coefficient estimated for human capital, which is proxied by the share of labor devoted to final good production, has a positive sign but is statistically insignificant. The re-estimation of the Solovianized Romer model using secondary enrollment rates to measure the impact of human capital on economic growth in Model 2 still produces a positive but statistically insignificant coefficient. In Model 3, human capital is proxied by secondary enrollment rates and the research intensity is proxied by the share of income devoted to the R&D. Third model reveals that the human capital has positive and statistically significant role on economic growth.[22]

[22] Even though a number of studies have found positive and significant contribution of human capital on economic growth, most of the studies suffer from the quality of the data and the

Table 3 reveals that the estimated coefficient of the technological growth, proxied by the share of R&D personnel in the labor force in Models 1 and 2, and the share of R&D expenditure in GDP in Model 3, is positive and significant. That is, the research intensity of a country has a crucial impact on its economic growth.

According to the specification tests reported in Table 3, the instruments are valid for the estimation of system GMM, which is clear from the Hansen test and the difference Hansen test. The p values relating to the first and second order serial correlations, given by M2, reject the existence of the serial correlation. In this respect, the overall performance of all models is good in terms of valid instrument selection and of expected signs and of the significance level of coefficients.

4 Concluding Remarks

The neoclassical growth model of Solow (1956) assumes that all countries face a common rate of technological progress. Following MRW (1992), the greater part of the convergence literature has assumed common rate of technical progress. We believe that the reason why the literature persists in this unrealistic assumption was the appropriateness of the convergence equation for empirical use. However, we argue that the main challenge was the development of a theory-based convergence equation which avoids this unrealistic assumption of constant and identical technological growth. In this respect, we combined the two strands of the growth literature, namely the neoclassical and the endogenous growth setups to develop a convergence equation that falsifies a constant and identical technological change across economies and in time.

The Solovianized Romer set up made it possible to avoid this assumption in the empirical part of our study. In particular, we re-estimated the MRW 1992 framework by using alternative proxies of R&D and human capital. We find that (i) the convergence rate is significant but lower than that suggested by the majority of previous studies, (ii) R&D, measured as a share of labor (income) devoted to R&D, has positive and significant impact on convergence (or

measurement error. In this context, Benhabib and Spiegel (1994) and Pritchet (1995) found that human capital may not have significant effect on output growth.

growth), (iii) human capital has positive and, in one definition, statistically significant contribution to convergence.

We argue that our study may have important implications for developing economies, though this work did not study it. If, as demonstrated, the convergence rate is really lower than that which the literature conjectures under constant and unchanging technological change assumption for a group of rather homogenous countries (e.g., OECD countries), then (the degree of) convergence between developing and developed economies is likely to be much lower. More than this, our framework logically suggests that not all developing economies will necessarily converge with developed economies. In particular, only those developing economies that allocate resources to R&D can be expected to have higher convergence rates. Consequently, we believe that there is an important need for further research in this direction.

References

Abramovitz M. (1986). "Catching Up, Forging Ahead, and Falling Behind". *Journal of Economic History,* 46(2), 385-406.

Aghion P. and P. Howitt (1992). "A Model of Growth through Creative Destruction". *Econometrica*, 60(2), 323-51.

Arellano, M. and O. Bover (1995). "Another look at the instrumental variable estimation of error-components models", Journal of Econometrics, 68 (1), 29-51.

Arrow, K.J. (1962). "The Economic Implications of Learning by Doing", Review of Economic Studies, 29 (3), 155–73.

Barro, R.J. (1990). "Government Spending in a Simple Model of Endogenous Growth". *Journal of Political Economy*, 98, S103-S125.

Barro, R.J. and J.W. Lee (2010), "A New Data Set of Educational Attainment in the World, 1950-2010", NBER Working Paper 15902.

Barro, R.J. and X. Sala-i-Martin (2003). *Economic Growth*, 2nd edition, MIT Press.

Baumol, W. J. (1986). "Productivity Growth, Convergence, and Welfare: What the Long-run Data Show". *American Economic Review*, 76(5), 1072-85.

Benhabib, J. and M. Spiegel (1994). "The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-country Data". *Journal of Monetary Economics*, 34, 143-173.

Bernard, A. B and C.I. Jones (1996). "Productivity across Industries and Countries: Time Series Theory and Evidence". The Review of Economics and Statistics, 78(1), 135-46.

Bloom D.E., Canning D. and J. Sevilla (2002). "Technological Diffusion, Conditional Convergence, and Economic Growth". NBER Working Paper 8713.

Blundell, R. and S. Bond. (1998). “Initial Conditions and Moment Restrictions in Dynamic Panel Data Models”. *Journal of Econometrics*, 87, 115–143.

Blundell, R. and S. Bond (2000). “GMM Estimation with Persistent Panel Data: An Application to Production Functions”. *Econometric Reviews*, 19(3), 321-340.

Blundell, R., S. Bond and F. Windmeijer (2000). “Estimation in Dynamic Panel Data Models: Improving on the Performance of the Standard GMM Estimator.” In B. Baltagi (ed.), *Nonstationary Panels, Panel Cointegration and Dynamic Panels*, Elsevier Science.

Bond, S., Hoeffler, A. and J. Temple (2001). GMM Estimation of Empirical Growth Models. *CEPR Discussion Paper No. 3048*.

Caselli, F., Esquivel G. and F. Lefort (1996). “Reopening the Convergence Debate: A New Look at Cross-Country Growth Empirics”. *Journal of Economic Growth*, 1, 363-389.

De Long B. (1988). “Productivity Growth, Convergence, and Welfare”. *American Economic Review*, 78(5), 1138-54.

Dowrick, S. and M. Rogers (2002). “Classical and Technological Convergence: Beyond the Solow-Swan Growth Model”. *Oxford Economic Papers*, 54(3), 369-385.

Durlauf, S.N., Kourtellos A. and C. M. Tan (2005). “Empirics of Growth and Development”, Discussion Papers Series, Department of Economics, Department of Economics, Tufts University.

Gong, G., A. Greiner and W. Semmler (2004). “Endogenous Growth: Estimating the Romer Model for the US and Germany”. *Oxford Bulletin of Economics and Statistics*, 66(2), 147-164.

Grossman G. and E. Helpman (1991). *Innovation and Growth in the Global Economy*, Cambridge, Mass., MIT Press.

Fagerberg, J. (1994). “Technology and International Differences in Growth Rates”, Journal of Economic Literature, 32 (3): 1147-1175.

Islam, N. (1995) “Growth Empirics: A Panel Data Approach”. *Quarterly Journal of Economics*, 110, 1127–70.

Jones, C. I. (1995), “R&D-Based Models of Economic Growth”. *Journal of Political Economy*, 103, 759–784.

Jones, C. (2002), “Sources of U.S. Economic Growth in a World of Ideas”. *American Economic Review*, 92, 220–39.

Ha, J., and P. Howitt (2007), “Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory”. *Journal of Money, Credit and Banking*, 39 (4), 733–774.

Hall, R. and C. Jones (1999). “Why Do Some Countries Produce So Much More Output per Workers than Others?”. *Quarterly Journal of Economics*, 114, 83–116.

Howitt, P. (2000).”Endogenous Growth and Cross-Country Income Differences”, *American Economic Review*, 90 (4), 829–846.

Keller, K. R. I. and P. Poutvaara (2005), “Growth in OECD Countries and Elsewhere: How Much Do Education and R&D Explain”. *Economics Bulletin*, 15, 1-11.

Kim, B. (2008). “Future of Economic Growth for South Korea”. *Asian Economic Journal*, 22, 397–410.

Kim, B. (2011). “Growth Regression Revisited: R&D Promotes Convergence?” forthcoming in *Applied Economics*.

Lucas R. (1988), “On the Mechanics of Economic Development.” *Journal of Monetary Economics*, 22, 3-42.

Madsen, J. B. (2008) “Semi-Endogenous versus Schumpeterian Growth Models: Testing the Knowledge Production Function Using International Data”. *Journal of Economic Growth*, 13(1), 1–26.

Mankiw, N. G., D. Romer, and D. N. Weil (1992) “A Contribution to the Empirics of Economic Growth”. *Quarterly Journal of Economics*, 107, 407-437.

McQuinn K. and K. Whelan (2007). “Conditional Convergence and the Dynamics of the Capital-output Ratio”. *Journal of Economic Growth*, 12, 159–184.

Murthy, N.R.V. and I.S. Chien (1997). “The Empirics of Economic Growth for OECD Countries: Some New Findings”. *Economics Letters*, 55, 425–29.

Nonneman, W., and P. Vanhoudt (1996) “A Further Augmentation of the Solow Model and the Empirics of Economic Growth for OECD Countries”. *Quarterly Journal of Economics*, 111, 943-953.

Petrakos, G., Arvanitidis P. and S.Pavleas (2007). “Determinants of Economic Growth: The Experts' View.” Dynamic Regions in a Knowledge Driven Global Economy Lessons and Policy Implications for the EU (DYNREG) Working Paper No. 20.

Pritchett, L. (1995). Where has all the education gone? World Bank Working Paper No. 1581.

Romer, P. M. (1986). “Increasing Returns and Long Run Growth”. *Journal of Political Economy*, 94, 1002–37.

Romer P. (1990), “Endogenous Technological Change”. *Journal of Political Economy* 98(5), S71-S102.

Roodman, D., (2006). How to do Xtabond2: An Introduction to ‘Difference’ and ‘System’ GMM in Stata. Center for Global Development Working Paper No. 103.

Sala-i-Martin, X. (1997), “I Just Ran Two Million Regressions.” *American Economic Review, Papers and Proceedings*, 87(2), 178-183.

Schneider, J. and T. Ziesemer (1995), “What's New and What's Old in New Growth Theory? Endogenous Technology, Microfoundation and Growth Rate Predictions - A Critical Overview”, Zeitschrift für Wirtschafts- und Sozialwissenschaften, 115(3), 1-44.

Solow R. (1956), “A Contribution to the Theory of Economic Growth”. *Quarterly Journal of Economics*, 70, 65-94.

Summers, R. and A. Heston, (1988), “A New Set of International Comparisons of Real Product and Price Levels Estimates for 130 Countries, 1950-1985”. *Review of Income and Wealth,* 34, 1-25.

Summers, R. and A. Heston (1991), “The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988”. *Quarterly Journal of Economics*, 106 (2), 327-368.

Windmeijer, F., (2005). “A Finite Sample Correction for the Variance of Linear Two-step GMM Estimators”. *Journal of Econometrics*, 126, 25–51.

Ulku H. (2004), “*R*&D Innovation and Economic Growth: An Empirical Analysis”. IMF Working Paper No. 185.

Young, A. (1994). “Lessons from the East Asian NICs: A Contrarian view”, European Economic Review, 38, 964-973.

Young, A. (1995). “The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience”, Quarterly Journal of Economics. 110, 641-680.

Zachariadis, M. (2004). “R&D-induced Growth in the OECD”. *Review of Development Economics*, 8, 423–39.

Annex A
Solovianized Romer Model with Skilled and Unskilled Labor

Suppose that the production technology is now:

$$Y = \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta} \cdot \int_0^{A(t)} X_i^{\alpha} di \qquad 0 < \alpha < 1 \qquad \text{(A.1)}$$

where Y is final good (GDP), $\overline{L}$ is the constant number of unskilled labor, H_Y is the number of human capital used in final good production, $1-\alpha-\beta$ and β are the respective production elasticities of unskilled and skilled labor, X_i are intermediate goods (varieties), and $A(t)$ is the number of intermediate goods at time t. We assume that there is perfect competition in the final-good sector and that final good is the numéraire. The profit equation is

$$\Pi_Y = \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta} \cdot \int_0^{A(t)} X_i^{\alpha} di - w_Y \cdot H_Y - w_L \cdot \overline{L} - \int_0^{A(t)} p_i \cdot X_i di \qquad \text{(A.2)}$$

In (A.2), w_Y and w_L are the real wage rates for the skilled and unskilled labor in final good sector, respectively. p_i is the user cost of intermediate-good i. Demand for each intermediate and human capital in the final-good production are then (we ignore demand for unskilled labor as its solution is a "residual" to the model for a given supply of unskilled labor):

$$\frac{\partial \Pi_Y}{\partial H_Y} = \beta \cdot \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta-1} \cdot \int_0^{A(t)} X_i^{\alpha} di - w_Y = 0 \qquad \text{(A.3a)}$$

$$\frac{\partial \Pi_Y}{\partial X_i} = \alpha \cdot \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta} \cdot X_i^{\alpha-1} - p_i = 0 \qquad \forall i \qquad \text{(A.3b)}$$

We continue to assume that the intermediate-good producing sectors only use 'raw' capital in order to produce an intermediate good: $K_i = X_i$ (or $k_i = x_i$ in per capita terms), where K_i measures the total amount of raw capital going into intermediate good of type i. Raw capital can be rented at the real rate of interest r plus depreciation δ: $r' \equiv r + \delta$. Hence, r' is the rental rate of capital. Additionally, we assume that each intermediate-good producer has monopoly power right over the production and sale of the good X_i. Therefore, the seller of the intermediate good faces a downward-sloping demand curve (cf., (A.3b)). Profit maximization of the i^{th} monopolist yields that price for the intermediate good i is $p_i = p = \frac{r'}{\alpha}$ and $X_i = X = \left(\frac{\alpha^2 \cdot \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta}}{r'} \right)^{\frac{1}{1-\alpha}}$. It is then straightforward to show that profits are identical across intermediates at a given time and that $K = \sum_{i=1}^{A(t)} K_i = \sum_{i=1}^{A(t)} X_i = X \cdot A$. This information implies $Y = \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta} \cdot K^{\alpha} \cdot A^{1-\alpha}$. Let us assume that the allocation of human capital is fixed at all times between final good and R&D sectors. If the knowledge production function is defined as before $\dot{A} = \eta \cdot H_{R\&D} \cdot A$, the growth rate of knowledge accumulation is $\hat{A} = \eta \cdot H_{R\&D} \equiv g$ and exogenous. Under perfectly competitive R&D sector assumption, equilibrium process implies $w_{R\&D} = V_{R\&D} \cdot \eta \cdot A$, where $V_{R\&D}$ is the value of patent. It is straightforward to show that the value of any patent is given by the arbitrage rule $V_{R\&D,i}(t) = \frac{\Pi(t)}{r(t)}$ and that the rule is valid as long as profit is constant, which is true at steady state. We continue to assume that the consumption-saving tradeoff is exogenous. In particular, capital accumulation is defined as

$$\dot{K} = s \cdot Y - \delta \cdot K \tag{A.4}$$

where K is capital, s is the saving (investment) rate, Y is output and δ is depreciation rate of capital. Recall that knowledge growth rate is $\hat{A} = \eta \cdot H_{R\&D}$ By using capital accumulation function, we can show that

$$\hat{\tilde{K}} = s \cdot \overline{L}^{1-\alpha-\beta} \cdot H_Y^{\beta} \cdot \tilde{K}^{\alpha-1} - (\delta + g) \tag{A.5}$$

where a tilde on top of a variable defines, $\tilde{K} = K / A$. Now suppose that we are at the steady state. It is well known that (A.5) implies

$$\tilde{K}_{ss} = \overline{L}^{\frac{1-\alpha-\beta}{1-\alpha}} \cdot H_Y^{\frac{\beta}{1-\alpha}} \cdot \left(\frac{s}{\delta + g} \right)^{\frac{1}{1-\alpha}} \tag{A.6}$$

Using (A.6) together with $X_{ss} = \tilde{K}_{ss}$ implies $r'_{ss} = \frac{\alpha^2(\delta + g)}{s}$. The rest of the model can be solved through substitution. In particular, one should be able to show that $\tilde{Y}_{ss} = \overline{L}^{\frac{1-\alpha-\beta}{1-\alpha}} \cdot H_Y^{\frac{\beta}{1-\alpha}} \cdot \left(\frac{s}{\delta + g} \right)^{\frac{\alpha}{1-\alpha}}$ and that $\hat{Y}_{ss} = \hat{K}_{ss} = \hat{A} = g$.

Determinants of Steady State Growth

The steady state value of output per capita becomes:

$$\frac{Y_{ss}}{\overline{L}+\overline{H}} = \left(\frac{L}{\overline{L}+\overline{H}}\right)^{\frac{\beta}{1-\alpha}} \cdot \left(\frac{H_Y}{\overline{L}+\overline{H}}\right)^{\frac{1-\alpha-\beta}{1-\alpha}} \cdot \left(\frac{s}{\delta+g}\right)^{\frac{\alpha}{1-\alpha}} \cdot A \qquad \text{(A.7)}$$

After some algebraic operations, we can show that

$$Ln[y_{ss}] = a + g\cdot t + \frac{\beta}{1-\alpha}Ln[\theta_L] + \frac{1-\alpha-\beta}{1-\alpha}Ln[\theta_Y] + \frac{\alpha}{1-\alpha}Ln[s] - \frac{\alpha}{1-\alpha}Ln[\delta+g] + \varepsilon \qquad \text{(A.8)}$$

where $y_{ss} = \frac{Y_{ss}}{\overline{L}+\overline{H}}$, $Ln[A(0)] = a + \varepsilon$, $g = \lambda \cdot H_{R\&D}$, $\theta_L = \frac{\overline{L}}{\overline{L}+\overline{H}}$ and $\theta_Y = \frac{H_Y}{\overline{L}+\overline{H}}$. Hence, θ_Y and $\theta_{R\&D}$ are the respective share of human capital in final good and R&D sector. Note that $\theta_Y + \theta_{R\&D} = 1$ for all times.

Convergence

Starting from (A.5), we may show that

$$\begin{aligned} Ln\left(\frac{Y(t)}{\overline{L}+\overline{H}}\right) - Ln\left(\frac{Y(0)}{\overline{L}+\overline{H}}\right) &= g\cdot t + \left(1-e^{-\nu\cdot t}\right)Ln\left(A(0)\right) \\ &- \left(1-e^{-\nu\cdot t}\right)Ln\left(\frac{Y(0)}{\overline{L}+\overline{H}}\right) + \left(1-e^{-\nu\cdot t}\right)\left(\frac{1-\alpha-\beta}{1-\alpha}\right)Ln(\theta_L) + \left(1-e^{-\nu\cdot t}\right)\left(\frac{\beta}{1-\alpha}\right)Ln(\theta_Y) \\ &+ \left(1-e^{-\nu\cdot t}\right)\left(\frac{\alpha}{1-\alpha}\right)Ln(s) - \left(1-e^{-\nu\cdot t}\right)\left(\frac{\alpha}{1-\alpha}\right)Ln(\delta+g) \end{aligned}$$

where $\nu = (1-\alpha)(\delta+g)$. That is,

$$Ln\left(\frac{Y(t)}{\overline{L}+\overline{H}}\right) - Ln\left(\frac{Y(0)}{\overline{L}+\overline{H}}\right) = \beta_0 + \beta_1 Ln\left(\frac{Y(0)}{\overline{L}+\overline{H}}\right) + \beta_2 Ln(\theta_L) + \beta_3 Ln(\theta_Y) + \beta_4 Ln(s) + \beta_5 Ln(\delta+g) \qquad \text{(A.9)}$$

where $\beta_0 = g\cdot t + \left(1-e^{-\nu\cdot t}\right)Ln\left(A(0)\right)$, $\beta_1 = -\left(1-e^{-\nu\cdot t}\right)$, $\beta_2 = \left(1-e^{-\nu\cdot t}\right)\left(\frac{1-\alpha-\beta}{1-\alpha}\right)$, $\beta_3 = \left(1-e^{-\nu\cdot t}\right)\left(\frac{\beta}{1-\alpha}\right)$, $\beta_4 = \left(1-e^{-\nu\cdot t}\right)\left(\frac{\alpha}{1-\alpha}\right)$ and $\beta_5 = -\beta_4$.

Annex B
Endogenous Allocation of Human capital in Solovianized Romer Model

We again assume an additively separable characteristic in the final good production function:

$$Y = H_Y^{1-\alpha} \sum_{i=1}^{A(t)} X_i^{\alpha} \qquad 0 < \alpha < 1 \qquad \text{(B.1)}$$

where H_Y is the number of human capital used in final good production, $1-\alpha$ is the respective production elasticity of that human capital, X_i are intermediate goods (varieties), and $A(t)$ is the number of intermediate goods at time t. We assume that the stock of human capital $\overline{H}$ is constant and that it is allocated between the final-good and R&D sectors endogenously. That is,

$$H_Y = \theta_Y(t) \cdot \overline{H} \text{ and } H_{R\&D} = \theta_{R\&D}(t) \cdot \overline{H} \qquad \text{(B.2)}$$

where H_Y ($H_{R\&D}$) is number of human capital employed by the final-good (R&D) sector, θ_Y ($\theta_{R\&D}$) is the respective share of human capital. Evidently, θ_Y and $\theta_{R\&D}$ are endogenous and that $\theta_Y(t) + \theta_{R\&D}(t) = 1$. First order profit maximization in the final-good and intermediate-good sectors are same. In return, we find that $x = x(\theta_Y(t), r(t))$ and that $\pi = \pi(\theta_Y(t), r(t))$. We continue to assume that $\hat{A} = \eta \cdot \theta_{R\&D}(t) \cdot \overline{H}$. The important difference is that $\hat{A}$ is now endogenous and that it has transitional dynamics, as the human capital allocation does have so. Equilibrium process in the R&D sector implies $V_{R\&D} \cdot \eta \cdot A = w_{R\&D}$. We are clearly able to show that the value of a patent is $V_{R\&D,i}(t) = \dfrac{\overline{H} \cdot \pi(t)}{r(t)}$ and that equilibrium process leads to $\dfrac{\overline{H} \cdot \pi(t)}{r(t)} \cdot \eta \cdot A = w_{R\&D}$. Given that at equilibrium the real wage must be same both in the R&D and final good market, $w_Y = w_{R\&D}$, we will have the following condition: $\theta_Y(t) = \dfrac{r(t)}{\alpha \cdot \eta \cdot \overline{H}}$. Under exogenous consumption-saving tradeoff, capital accumulation rule would be $\dot{K} = s \cdot Y - \delta \cdot K$.

Steady-state Analysis

We will now prove formally show that $\hat{y}_{ss} = \hat{k}_{ss} = \hat{A}_{ss}$ holds. First of all, from physical capital accumulation, it is straightforward to show that

$\tilde{k}_{ss} = \theta_{Y,SS} \cdot \left(\frac{s}{\delta + \hat{A}_{ss}} \right)^{\frac{1}{1-\alpha}}$. Secondly, recall that $x_{ss} = \theta_{Y,ss} \left(\frac{\alpha^2}{r'_{ss}} \right)^{\frac{1}{1-\alpha}}$. Given that $\tilde{k}_{ss} = x_{ss}$ must hold, it is easy to show that $\frac{s}{\delta + \hat{A}_{ss}} = \frac{\alpha^2}{r'_{ss}} \Rightarrow$ $s \cdot r'_{ss} = \alpha \cdot \delta + \alpha \cdot \eta \cdot \theta_{R\&D,SS} \cdot \overline{H}$. Thirdly, using $\theta_{Y,ss} = \frac{r_{ss}}{\alpha \cdot \eta \cdot \overline{H}}$ and $\theta_{Y,ss} + \theta_{R\&D,ss} = 1$, we can show that $r_{ss} = \frac{(\alpha - s)\delta + \alpha \cdot \eta \cdot \overline{H}}{1+s}$. The rest follows. For example, $\theta_{Y,ss} = \frac{(\alpha - s)\delta + \alpha \cdot \eta \cdot \overline{H}}{(1+s) \cdot \alpha \cdot \eta \cdot \overline{H}}$. it is easy to find that $\hat{y}_{ss} = \hat{k}_{ss} = \hat{A}_{ss} = \frac{s \cdot \alpha \cdot \eta \cdot \overline{H} - (\alpha - s)\delta}{\alpha(1+s)} \equiv g$ (we assume that $s > \frac{\alpha\delta}{\alpha \cdot \eta \cdot \overline{H} + \delta}$). It is notable that higher saving rates and higher productivity in R&D have a positive impact on the growth rate, as expected *a priori*. The model, however, does not fit very well to empirical analysis.

Determinants of Steady State Growth

Recall that we found the steady state value of output per efficient capita as $\tilde{y}_{ss} = \theta_{Y,SS} \cdot \left(\frac{s}{\delta + \hat{A}_{ss}} \right)^{\frac{\alpha}{1-\alpha}}$. By taking natural log of it, and after a simple algebraic transformation, we can show that

$$\ln y_{ss} = a + g \cdot t + \ln[\theta_{Y,SS}] + \frac{\alpha}{1-\alpha} \ln[s] - \frac{\alpha}{1-\alpha} \ln[\delta + g] + \varepsilon \qquad \text{(B.3)}$$

where $Ln[A(0)] = a + \varepsilon$. Notably, $\theta_{Y,SS}$ and g are function of a bunch of parameters that may not always be useful in empirical analysis.

Transitional Dynamics

Here are the equations derived from the model:

$$k(t) = x(t) \cdot A(t)$$

$$\theta_Y(t) + \theta_{R\&D}(t) = 1$$

$$\hat{A}(t) = \eta \cdot \theta_{R\&D}(t) \cdot \overline{H}$$

$$\theta_Y(t) = \frac{r(t)}{\alpha \cdot \eta \cdot \overline{H}}$$

$$x(t) = \theta_Y(t) \cdot \left(\frac{\alpha^2}{r'(t)} \right)^{\frac{1}{1-\alpha}}$$

$$\dot{k}(t) / k(t) = s \cdot (\theta_Y(t))^{1-\alpha} \cdot (k(t))^{\alpha-1} \cdot (A(t))^{1-\alpha} - \delta$$

Through substitution, we may easily show, for example, that

$$\frac{\dot{r}}{r} = \frac{(1-\alpha)(r(t)+\delta)}{\delta - \alpha \cdot r(t) - \alpha\delta} \frac{(s+\alpha)\cdot r(t) + s\delta - \delta\alpha^2 - \alpha^2 \eta \cdot \overline{H})}{\alpha^2}$$

Unfortunately solving this nonlinear differential equation is not a straightforward task. Hence, we fail to generate a convergence equation.

Annex C
Generalized Knowledge Production Function
(Jones (2002) under Exogenous Allocation of Human capital)

We assume that the production technology has an additively separable characteristic:

$$Y = H_Y^{1-\alpha} \sum_{i=1}^{A(t)} X_i^{\alpha} \qquad 0 < \alpha < 1 \qquad \text{(C.1)}$$

where H_Y is the number of human capital used in final good production, $1-\alpha$ is the respective production elasticity of that human capital, X_i are intermediate goods (varieties), and $A(t)$ is the number of intermediate goods at time t. Our model in this version deviates from Romer (1990) in three respects. First, the growth and allocation of the stock of human capital is now different. We assume that human capital grows at rate n. On the other hand, we continue to assume that the allocation of human capital between competing sectors is exogenous:

$$H_Y = \theta_Y \cdot H \text{ and } H_{R\&D} = \theta_{R\&D} \cdot H \qquad \text{(C.2)}$$

where H_Y ($H_{R\&D}$) is number of human capital employed by the final-good (R&D) sector, θ_Y ($\theta_{R\&D}$) is the share of human capital engaged in final-good (R&D) sector, and H is the stock of human capital. θ_Y and $\theta_{R\&D}$ are exogenously determined and their sum is unity. Obviously, H_Y and $H_{R\&D}$ also grow at rate n. In per capita terms, the production function implies $y = \theta_Y^{1-\alpha} \sum_{i=1}^{A(t)} x_i^{\alpha}$, where x_i are intermediate goods in per capita.

Second, the consumption-saving tradeoff is exogenous, á la Solow (1956). We call this Solovianization of Romer (1990). This assumption implies that capital accumulation is

$$\dot{K} = s \cdot Y - \delta \cdot K \qquad \text{(C.3)}$$

where K is capital, s is the saving (investment) rate, Y is output and δ is depreciation rate of capital. Obviously, equation (C.3), expressed in per capita terms, is $\dot{k} = s \cdot y - (n+\delta) \cdot k$.

Third, we deviate from Romer (1990) by employing a generalized R&D (knowledge) production function:

$$\dot{A} = \lambda \cdot H_{R\&D}^{\beta} \cdot A^{\phi} \qquad \beta \leq 1,\ \phi \leq 1 \qquad \text{(C.4)}$$

Knowledge (patent) production $\dot{A}$ depends positively on the number of human capital (=researchers) and the existing stock of patents, á la Romer. β is production elasticity of $H_{R\&D}$ and ϕ indicates the degree of positive externality. Deviating from Romer (1990), we assume that productivity of researchers in R&D sector is subject to diminishing marginal productivity. Note that β determines the speed of this diminishing marginal productivity. Similarly, we assume that the existing stock of patents, which is a positive externality in the model, is subject to diminishing marginal productivity. This latter affect is often called the "giants shoulders" effect. Recall that in Romer (1990), $\beta = 1$ and $\phi = 1$. Let us now give briefly the solution of the generalized Romer model.

Final Good Sector

We assume that there is perfect competition in the final-good sector and we take final output to be the numéraire. Hence, the profit equation is

$$\Pi_Y = H \cdot \left[\theta_Y^{1-\alpha} \sum_{i=1}^{A(t)} x_i^{\alpha} - w_Y \theta_Y - \sum_{i=1}^{A(t)} p_i x_i \right] \qquad \text{(C.5)}$$

where w_Y is the real wage rate and p_i is the user cost of intermediate-good i. First order conditions for profit maximization are as follows:

$$\frac{\partial \Pi_Y}{\partial H_Y} = (1-\alpha)\theta_Y^{-\alpha} \sum_{i=1}^{A(t)} x_i^{\alpha} - w_Y = 0 \qquad \text{(C.6a)}$$

$$\frac{\partial \Pi_Y}{\partial X_i} = \alpha \theta_Y^{1-\alpha} x_i^{\alpha-1} - p_i = 0 \qquad \forall i \qquad \text{(C.6b)}$$

The above-given first-order profit maximization with respect to H_Y and X_i are inverse demand functions for human capital (employed at final-good production) and any individual intermediate i.

Intermediate-good Sector

We continue to assume that the intermediate-good producing sectors only use 'raw' capital in order to produce an intermediate good: $K_i = X_i$ (or $k_i = x_i$ in per capita terms), where K_i measures the total amount of raw capital going into intermediate good of type i. Raw capital can be rented at the real rate of interest r plus depreciation δ: $r'(t) \equiv r(t) + \delta$. Hence, $r'(t)$ is the rental rate of capital. We assume that each intermediate-good producer has monopoly power right over the production and sale of the good X_i, as the patent (the blueprint) of the product belongs to it. Hence, the seller of the intermediate good faces a downward-sloping demand curve (cf., equation (C.6b)). Therefore, the profit that the i^{th} monopolist has to maximize is:

$$\Pi_i = H \cdot \left[p_i \cdot x_i - r' \cdot x_i \right] \qquad \text{(C.7)}$$

The profit maximizing price for the intermediate good i is obtained as $p_i = p = \frac{r'}{\alpha}$, that is, price is identical across intermediates. Note that substitution of price information in equation (C.6b) reveals that $x_i = x = \theta_Y \left(\frac{\alpha^2}{r'} \right)^{\frac{1}{1-\alpha}}$, the quantity of each intermediate are identical across varieties. Using this information in (C.7) implies $\frac{\Pi_i}{H} = \left(\frac{1-\alpha}{a} \right) \cdot r' \cdot x = (1-\alpha)\alpha\theta_Y^{1-\alpha} x^{\alpha}$. Hence, profits are also identical across intermediates at a given time (both in per capita and levels). Given our findings that price, quantity and profit are identical across intermediates, it must be true that

$$K = \sum_{i=1}^{A(t)} K_i = \sum_{i=1}^{A(t)} X_i = X \cdot A \quad \text{or} \quad k = \sum_{i=1}^{A(t)} k_i = \sum_{i=1}^{A(t)} x_i = x \cdot A \qquad \text{(C.8a)}$$

$$y = \theta_Y^{1-\alpha} \cdot x^{\alpha} \cdot A \text{ or } y = \theta_Y^{1-\alpha} \cdot k^{\alpha} \cdot A^{1-\alpha} \qquad \text{(C.8b)}$$

These are modified Romer (1990) results due to fixed allocation of labor.

R&D Sector

We assume that the R&D sector is a perfectly competitive sector. Given the knowledge production function at (5), the profit equation is

$$\Pi_{R\&D} = V_{R\&D} \cdot \lambda \cdot H_{R\&D}^{\beta} \cdot A^{\phi} - w_{R\&D} \cdot H_{R\&D} \qquad \text{(C.9)}$$

where $V_{R\&D}$ is the value of patent. First-order profit maximization implies $V_{R\&D} \cdot \lambda \cdot \beta \cdot H_{R\&D}^{\beta-1} \cdot A^{\phi} = w_{R\&D}$. We need to express $V_{R\&D}$ in a simpler way in order to proceed further. To this end, note that the present value of all profits derived from any patent (blueprint), e.g., $V_{R\&D,i}$ is given by:

$$V_{R\&D,i}(t) = \int_t^{\infty} \pi \cdot L \cdot e^{-\int_t^{\tau} r(s)ds} d\tau \Rightarrow V_{R\&D,i}(t) = \frac{\pi(t)}{r(t)-n} \qquad \text{(C.10)}$$

where $\Pi_i / H = \pi_i = \pi$. In (11), $\left[r(t) - n \right] \cdot V_{R\&D,i} = \pi(t)$ is an arbitrage rule. The rule is valid as long as the real rate of interest r is constant at the steady state (recall that $x = x(r)$ and $\pi = \pi(r)$).

If we go back to first-order profit maximization in R&D sector, $V_{R\&D} \cdot \lambda \cdot L_{R\&D}^{\beta} \cdot A^{\phi} = w_{R\&D}$ becomes $\frac{\pi(t)}{r(t)-n} \cdot \lambda \cdot L_{R\&D}^{\beta} \cdot A^{\phi} = w_{R\&D}$ (recall that $w_{R\&D} = w_Y$ is not valid under exogenous allocation of human capital).

Steady-state Analysis

Firstly, from (C.4), we can show that $\hat{A}_{ss} = \frac{\beta \cdot n}{1-\phi}$. Hence, the degree of externality has a positive contribution on the growth rate of knowledge accumulation. Secondly, from capital accumulation, $\dot{k} = s \cdot y - (n+\delta) \cdot k$, we can show that $\hat{k}_{ss} = \hat{A}_{ss}$. Third, using production function $y = s \cdot \theta_Y^{1-\alpha} \cdot k^{\alpha} \cdot A^{1-\alpha}$, it is easy to get that $\hat{y}_{ss} = \hat{k}_{ss} = \hat{A}_{ss}$. Fourth, steady state value of capital per efficient skilled labor, $\tilde{k} = \frac{K}{A \cdot H}$, can be determined from the capital accumulation as $\tilde{k}_{ss} = \theta_Y \cdot \left(\frac{s}{n+\delta+\hat{A}} \right)^{\frac{1}{1-\alpha}}$ or $k_{ss} = \theta_Y \cdot A_{ss} \cdot \left(\frac{s}{n+\delta+\frac{\beta \cdot n}{1-\phi}} \right)^{\frac{1}{1-\alpha}}$ (see below for definition of A_{ss}). Notably, A_{ss} and k_{ss} are approaching infinity, as expected. The behavior of output per skilled worker along the steady state follows capital per skilled worker:

$$y_{ss} = \theta_Y \cdot A_{ss} \cdot \left(\frac{s}{n+\delta+\frac{\beta \cdot n}{1-\phi}} \right)^{\frac{\alpha}{1-\alpha}} \quad \text{or} \quad \tilde{y}_{ss} = \theta_Y \cdot \left(\frac{s}{n+\delta+\frac{\beta \cdot n}{1-\phi}} \right)^{\frac{\alpha}{1-\alpha}} .$$

The steady state value of the real interest rate can be determined through $k_{ss} = x_{ss} \cdot A_{ss}$: $r'_{ss} = \frac{n+\delta+\alpha^2 \frac{\beta \cdot n}{1-\phi}}{s}$. The rest follows.

Determining $A(t)$ and $\hat{A}(t)$

Let us now find $A(t)$ and $\hat{A}(t)$. We need this information for determining the long run determinants of economic growth and convergence equation. When knowledge accumulation is subject to transitional dynamics, it is not anymore possible to utilize the standard derivations. We may use the knowledge accumulation function in order to determine the time path of knowledge stock. To this end,

$$\dot{A} = \lambda \cdot H_{R\&D}^{\beta} \cdot A^{\phi} \Rightarrow \; A(t) = \left[\frac{(1-\phi)\cdot \lambda \cdot (\theta_{R\&D} \cdot H)^{\beta}}{\beta \cdot n} + (1-\phi)\cdot const \right]^{\frac{1}{1-\phi}}.$$

Obviously, $const = \frac{(A(0))^{1-\phi}}{1-\phi} - \left(\frac{\lambda \cdot \theta_{R\&D}^{\beta}}{\beta \cdot n} \right)$. You may easily show that $Lim_{t\to\infty} A(t) = \infty$. Note also that

$$Lim_{t\to\infty} A(t) = Lim_{t\to\infty} e^{\frac{\beta \cdot n}{1-\phi}t} \cdot Lim_{t\to\infty} \left[\frac{(1-\phi)\cdot \lambda \cdot \theta_{R\&D}^{\beta}}{\beta \cdot n} + (1-\phi)\cdot const \cdot e^{-\left(\frac{\beta \cdot n}{1-\phi}\right)t} \right]^{\frac{1}{1-\phi}}$$

$$A_{ss} \equiv Lim_{t\to\infty} A(t) = e^{\frac{\beta \cdot n}{1-\phi}t} \cdot \left[\frac{(1-\phi)\cdot \lambda \cdot \theta_{R\&D}^{\beta}}{\beta \cdot n} \right]^{\frac{1}{1-\phi}}.$$

That is, we may legitimately ignore the constant at the steady state. It is also straightforward to show that $\frac{\dot{A}(t)}{A(t)} = \left(\frac{\beta \cdot n}{1-\phi} \right) \left(\frac{1}{1 + \frac{const \cdot \beta \cdot n}{\lambda \cdot \theta_{R\&D}^{\beta}} e^{-\beta \cdot n \cdot t}} \right)$ and that $Lim_{t\to\infty} \dot{A}/A \equiv \hat{A}_{ss} = \frac{\beta \cdot n}{1-\phi}$.

Determinants of Steady State Growth

Let us first derive an equation that determines the sources of growth at the steady state. Recall that the steady state value of output per skilled worker is:

$$y_{ss} = \theta_Y \cdot e^{\frac{\beta \cdot n}{1-\phi}t} \cdot \left[\frac{(1-\phi)\cdot \lambda \cdot \theta_{R\&D}^{\beta}}{\beta \cdot n} \right]^{\frac{1}{1-\phi}} \cdot \left(\frac{s}{n + \delta + \frac{\beta \cdot n}{1-\phi}} \right)^{\frac{\alpha}{1-\alpha}} \qquad \text{(C.11)}$$

By taking natural log of equation (C.11), we can show that

$$\ln y_{ss} = \frac{\alpha}{1-\alpha}\ln[s] + \ln[\theta_Y] + \frac{1}{1-\phi}\ln[\lambda] + \frac{\beta}{1-\phi}\ln[\theta_{R\&D}] - \frac{1}{1-\phi}\ln[g]$$
$$- \frac{\alpha}{1-\alpha}\ln[g + n + \delta] + gt \qquad \text{(C.12)}$$

where $g = \frac{\beta \cdot n}{1-\phi}$ and $H(0) = 1$ for simplicity.

Convergence

The derivation of convergence equation requires one additional simplification for algebraic tractability, to our best knowledge. In particular, we need to assume that $\phi = 1-\beta$, which implies a constant returns to scale knowledge production function: $\dot{A} = \lambda \cdot H_{R\&D}^{\beta} \cdot A^{1-\beta}$. We need to make this assumption in order to transform the function into per skilled worker, which is necessary for a steady state solution. Under this restriction, if we divide both sides by the stock of human capital, the knowledge production function becomes $\frac{\dot{A}}{A} = \lambda \cdot H_{R\&D}^{\beta} \cdot A^{-\beta}$, which implies $g \equiv \hat{A}_{ss} = n$. In per capita terms, knowledge production function becomes

$$\dot{a} = \lambda \cdot \theta_{R\&D}^{\beta} \cdot a^{1-\beta} - n \cdot a \qquad \text{(C.13)}$$

Notably, the function has a steady state value: $a_{ss} = \left(\frac{\lambda}{n}\right)^{\frac{1}{\beta}} \cdot \theta_{R\&D}$. We have already shown that $\tilde{y} = \theta_Y^{1-\alpha} \cdot \tilde{k}^{\alpha}$ and that capital accumulation function in per efficient skilled worker is $\dot{\tilde{k}} = s \cdot \theta_Y^{1-\alpha} \cdot \tilde{k}^{\alpha} - (n + \delta + \hat{A}(t)) \cdot \tilde{k}$, which implies $\tilde{k}_{ss} = \theta_Y \cdot \left(\frac{s}{n+\delta+g}\right)^{\frac{1}{1-\alpha}}$. Since there are two equations of motion, we need to linearize them together. To this end, let us define,

$$m = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} Ln(\tilde{k}) \\ Ln(a) \end{bmatrix}, \ \dot{m} = \begin{bmatrix} \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} \frac{dLn(\tilde{k})}{dt} \\ \frac{dLn(a)}{dt} \end{bmatrix},$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} -(1-\alpha)(n+\delta+g) & \beta n \\ 0 & -\beta n \end{bmatrix}$$

$$m_{ss} = -\begin{bmatrix} m_{1,ss} \\ m_{2,ss} \end{bmatrix} = -\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} Ln(\tilde{k}_{ss}) \\ Ln(a_{ss}) \end{bmatrix}$$

The linearized differential equation system around steady state values of $\tilde{k}$ and a are:

$$\begin{bmatrix} \frac{dLn(\tilde{k})}{dt} \\ \frac{dLn(a)}{dt} \end{bmatrix} = \begin{bmatrix} (1-\alpha) \cdot s \cdot \theta_Y^{1-\alpha} \cdot \tilde{k}_{ss}^{\alpha-1} & \beta \cdot \lambda \cdot \theta_{R\&D}^{\beta} \cdot a_{ss}^{-\beta} \\ 0 & -\beta \cdot \lambda \cdot \theta_{R\&D}^{\beta} \cdot a_{ss}^{-\beta} \end{bmatrix} \begin{bmatrix} Ln(\tilde{k}) - Ln(\tilde{k}_{ss}) \\ Ln(a) - Ln(a_{ss}) \end{bmatrix},$$

which can be compactly denoted as

$$\dot{m} = B \cdot m + m_{ss} \qquad \text{(C.14)}$$

Suppose that there is an invertible nxn vector V, which satisfies $m = V \cdot z$. Multiplying both sides of (C.14) by V^{-1} and defining $D = V^{-1} \cdot A \cdot V$ and $z_{ss} = V^{-1} \cdot m_{ss}$ yields

$$\dot{z} = D \cdot z + z_{ss} \qquad \text{(C.15)}$$

It is well-known that A and D have same eigenvalues. It is easy to show that eigenvalues of the system are $d_1 = -(1-\alpha)(n+\delta+g)$ and $d_2 = -\beta \cdot n$ and that eigenvectors are $V_1 = \begin{bmatrix} 0 & \beta n \\ 0 & -\beta n + (1-\alpha)(n+\delta+g) \end{bmatrix}$ and $V_2 = \begin{bmatrix} \beta n - (1-\alpha)(n+\delta+g) & \beta n \\ 0 & 0 \end{bmatrix}$. Hence, $V = \begin{bmatrix} 1 & 1 \\ 0 & \dfrac{-\beta n + (1-\alpha)(n+\delta+g)}{\beta n} \end{bmatrix}$ and $V^{-1} = \left(\dfrac{-\beta n + (1-\alpha)(n+\delta+g)}{\beta n} \right)^{-1} \begin{bmatrix} \dfrac{-\beta n + (1-\alpha)(n+\delta+g)}{\beta n} & -1 \\ 0 & 1 \end{bmatrix}$.

Clearly, (C.15) becomes easy to solve by using the integrating factor method. In particular, one may show that[23]

$$z_1 = \frac{b_1}{(1-\alpha)(n+\delta+g)} + const_1 \cdot e^{-(1-\alpha)(n+\delta+g)t}$$
$$z_2 = \frac{b_2}{\beta n} + const_2 \cdot e^{-\beta n t}$$

where

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} (1-\alpha)(n+\delta+g)\beta n & \dfrac{-(1-\alpha)(n+\delta+g)\beta n}{(1-\alpha)(n+\delta+g)-\beta n} \\ 0 & \dfrac{\beta n \cdot \beta n}{(1-\alpha)(n+\delta+g)-\beta n} \end{bmatrix} \begin{bmatrix} Ln(\theta_Y) + \dfrac{1}{1-\alpha} Ln(\dfrac{s}{n+\delta+g}) \\ Ln(\theta_{R\&D}) + \dfrac{1}{\beta} Ln(\dfrac{\lambda}{n}) \end{bmatrix}$$

Since $m = V \cdot z$, we can easily show that

$$Ln(\tilde{k}) = \frac{b_1}{(1-\alpha)(n+\delta+g)} + const_1 \cdot e^{-(1-\alpha)(n+\delta+g)t} + \frac{b_2}{\beta n} + const_2 \cdot e^{-\beta n t}$$
$$Ln(a) = \frac{(1-\alpha)(n+\delta+g)-\beta n}{\beta n} \left(\frac{b_2}{\beta n} + const_2 \cdot e^{-\beta n t} \right)$$

---

[23] We do not show $const_1$ and $const_2$ for matter of simplicity.

Recalling $Ln(A) = Ln(a) + nt$, $Ln(\tilde{y}) = Ln(y) - Ln(A)$ and that $Ln(\tilde{y}) = (1-\alpha)Ln(\theta_Y) + \alpha Ln(\tilde{k})$, we may get

$$Ln(y) = (1-\alpha)Ln(\theta_Y) + \alpha Ln(\tilde{k}) + Ln(a) + nt \qquad \text{(C.16)}$$

This equation can be used in empirical analysis.

Annex D
List of OECD Countries

Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.