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Learning by Observing

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Abstract

We introduce a network formation model based on the idea that individuals engage in production (or decide to participate in an action) depending on the similar actions of the people they observe in the society. We differentiate from the classical models of participation by letting individuals to choose, non-cooperatively, which agents to observe. Observing behavior of others is a costly activity but provides benefits in terms of reduction in cost of production for the observing agent, which we take it as learning.

In this non-cooperative setting we provide complete characterization of both Nash stable and socially efficient network configurations. We show that every society can admit a stable network. Moreover, typically there will be multiple stable configurations that will be available for a society. While all stable networks will not be efficient, we show that every efficient network will be stable.

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1 Introduction

This paper aims to provide a perspective for understanding how actions of agents in groups are influenced through interactions with other members of a society. We acknowledge the rich literature both in game theory, and social networks (participation games, Networks) that focuses similar topics.

Here we pick a very specific interaction. We consider situations where one's decision to engage in an action, such as a social movement, adapting a fashion or religion, a production activity, or taking a portfolio decision depends on how people around one's social circle behave. Observing people taking similar actions somehow reduces one's *cost* of engaging in the same action. In the context of a production decision, this reduction in cost may be due to productivity gains as a result of learning a technique. For an artist, observing works of others should be inspiring, leading higher chances of better output, (from this perspective it is not surprising that we see artists generally reside in close neighborhoods where such observations are easier.) In the context of adapting a fashion, religion participating a social movement, observing people making similar choices helps you to feel that you are not a complete outlier. Similarly, for a researcher, subscribing journals to follow works of others is obviously productivity enhancing and is necessary for creation of original work. We shall refer this effect as *learning* (through observing). There is also a vast literature on learning. ([1],[2],[5],[3],[6],[4])

A common and critical property of above examples is that one is free to *choose* whom to observe. It is generally the case that observing all members of a society in not feasible, otherwise prohibitively costly. So one must *choose* a subset of agents to observe from overall society. This differentiates the situations we focus from situations where you have a fixed neighborhood structure whose overall behavior influences you (see, for example [1]). Through this differentiation we can also provide insights not only the actions of agents but also about the networks that will form from such observing behavior of agents in a society. This will allow us to identify influencing groups of agents in a society, i.e. hubs.

In this paper we look at the structural properties of societies with above characteristics. More specifically following questions are of interest and shall be addressed. (1) Which configurations of engagement and communication structures emerge in societies with agents that are motivated by self interest? (2) How societies that are organized by self interest compare with the socially efficient configurations?

Inspired by the situations described, we develop a model to provide formal framework to answer above questions. We consider societies in which members take two strategic decisions simultaneously; whether to engage in an action or not and the subset of agents they will observe in the society. The cost benefit trade off of these two actions are as follows. Engaging in an action provides a fixed benefit to the agent but is also costly. This cost, which we shall refer as *engagement cost*, can be reduced if one is able to observe other agents that are also engaged in the action. That is, the more people you observe that are also engaged in an action, the more you learn, and is less costly for you to engage. However, observing others is also costly to the agent. So although engaging in an action provides benefits, the final payoff depends on cost of engagement and total cost of observing others. It is possible to have situations where an agent wishes to engage in an action, but cannot, since, in case she does, she will not be able find sufficient number of people that she can observe to reduce engagement cost sufficiently. But once engagement decision is given agent increases its observation set as long as the marginal reduction in cost of engagement is higher than the cost of observing the last included agent.

The observing behavior of agents can be represented with a network where nodes correspond to agents and an edge from node i to j means, i observes j. Since i's observation of j does not imply that j observes i, the network is *directed*.

In this setting we first focus on the stable configurations of the non-cooperative simultaneous move normal form game. We provide a complete characterization of stable network configurations a society can admit under different cost structures. As stability notion we employ the Nash equilibrium. So a society is considered stable if for each agent the proposed strategy is her best response to the state of the rest of the society. Then we provide a characterization of societies that are efficient, i.e. that maximize that total payoff. We then compare the two characterizations.

We interprete our formal results as follows. Since our characterizations cover all possibilities, it follows that there exists a stable state for every society to admit. The configuration of this state, expectedly, depends on the cost structure.

When cost of engagement is high, there are two possibilities. First, there is no possibility of engagement since the costs are prohibitively high, and the society naturally produces nothing. Second, engagement is feasible but inducing an agent to engage in an action requires high number other engaging agents to be observed. Hence existence of engaging agents in such societies requires coordination among many agents. Otherwise it is easy for a society to get stuck in a state where no agent acts, unless, for some reason the collective action is somehow stimulated.

On the other extreme, when the engagement cost is low, learning is very rapid through observing just a few agents, or cost of observing others is very low, then every agent engages and observe payoff maximizing number of agents in the society. As agents are symmetric, number of people each agent observes must be almost the same. When production cost is low but cost of observing others is high, it is possible that everybody engages but observe no one.

In the intermediate cases, we observe societies with partial engagement. Yet this is a very rare case as the setting requires symmetric payoff between both agents that produce and agents that don't.

The paper is organized as follows. Section 2 introduces the formal model. We provide the results on stability in section 3. This is followed by efficiency characterization in section 4. In section 5 we provide an example to illustrate results of the paper. Section 6 concludes.

2 A Model of Learning by Observing

We denote by $N = \{1, \ldots, n\}$ the set of agents in a society. Each agent $i \in N$ decides on two issues: whether to engage in production of non-rival good ($y_i = 1$) or not ($y_i = 0$) and the set of agents, S_i, that she wishes to observe. A *strategy* of agent i is, therefore, a pair (S_i, y_i) where $S_i \subset N \setminus \{i\}$ and $y_i \in \{0, 1\}$. Define $S = \{S_1, \ldots, S_n\}$ as a *network* and $y = \{y_1, \ldots, y_n\}$ as a *production configuration.* An n-tuple of strategies $(S, y) = \{(S_i, y_i)\}_{i \in N}$ is called a *network configuration.*

Observing each agent in S_i, costs $\gamma > 0$ for agent i. The benefit of observing other producers is due to reduction in cost of production. Let $\beta[]$ be the cost of engaging in production. If no agent is observed then this cost is at its maximum, $\bar{\beta} = \beta[0] > 0$. With each observed producer, this cost decreases, yet, with a decreasing rate. Hence we require $\beta'[] < 0, \beta''[] > 0$. The lower bound of the cost is$\underline{\beta} = lim_{x \to \infty} \beta[x] \geq 0$. The payoff function of agent i in network configuration (S, y) is given by

$$\pi_i[S, y] = y_i - \beta\Big[\sum_{j \in S_i} y_j\Big] y_i - \gamma |S_i|$$

In what follows we will be focusing on the properties of the pure strategy Nash equilibrium of the normal form game $(N, \{\Sigma_i\}_{i \in N}, \{\pi_i\}_{i \in N})$ where Σ_i is i's strategy space $\Sigma_i = \{(S_i, y_i) | S_i \subset N \setminus \{i\}, y_i \in \{0, 1\}\}$.

3 Stable Network Configurations

Let T be the set of number of agents that are necessary to provide a non-negative payoff from engaging in production. Formally, $T \subset \{0, 1, \ldots, n-1\}$ such that $\forall i \in T, 1 - \beta[i] - \gamma i \geq 0$. We shall refer to T as *engagement inducement* set. Let $t^* \subset T$ be the set of number of agents that one needs to observe to maximize her payoff in the case that she engages in production, i.e. $t^* = \{i \in T | \forall j \neq i, \beta[j] + \gamma j \geq \beta[i] + \gamma i\}$. Let $\tilde{t}$ be the set of minimum number of producing agents that is required to induce an agent to produce. Formally, let $i \in T$ be such that $\forall j \in T \setminus \{i\}, j > i$. Then define $\tilde{t} = \{i, i+1\}$ if $i+1 \in T$ and $1 - \beta[i] - \gamma i = 1 - \beta[i+1] - \gamma(i+1)$ otherwise $\tilde{t} = \{i\}$.

The strict concavity of the cost function implies that if $1 - \beta[i] - \gamma i = 1 - \beta[i+1] - \gamma(i+1)$, we cannot have $1 + \beta[i+2] - \gamma(i+2) = 1 - \beta[i+1] - \gamma(i+1)$, since $\beta[i] - \beta[i+1] > \beta[i+1] - \beta[i+2]$. Therefore the sets $\tilde{t}$ and t^* can have at most two members.[1]

We start with characterization of societies depending on production configuration. In societies where no agent engages in production, the unique network structure is the empty network, since cost benefits cannot be realized (both because agents do not produce and there are no producing agents to observe). Our first result states that when production inducement set is empty, the unique stable network configuration is the empty network with no production.

Proposition 1. *If $T = \emptyset$, then the unique Nash network configuration (S, y) is the empty network with no production, i.e. $\forall i \in N, S_i = \emptyset, y_i = 0$.*

Proof of Proposition 1. Suppose $T = \emptyset$. If (S, y) is a configuration with no production then no agent forms any link (as links are costly), so it must an empty network. If $\exists i \in N$ with $y_i = 1$, then it must be that $1 - \beta[|S_i|] - \gamma|S_i| \geq 0$ contradicting with the assumption $T = \emptyset$. □

When production inducement set is non empty, it is always possible to construct a society in which all agents engage in production. Unlike the case of no production, production configuration itself, however, does not necessarily imply a unique network structure. While it is possible to have networks where agents observe each other, i.e. a complete network, there are also other possibilities. Suppose, for example, $T = \{0\}$, in which case, a society where all agents produce but none observe (empty network) is stable. The following result provides the necessary and sufficient condition for existence of an all producing society.

Proposition 2. *There always exists a Nash network configuration where $\forall i \in N, y_i = 1$ if and only if $T \neq \emptyset$.*

Proof of Proposition 2. Suppose $T \neq \emptyset$. Define (S, y) such that $\forall i \in N, y_i = 1, |S_i| \in t^*$. By definition of t^*, observing $|S_i|$ agents maximizes the payoff of typical agent i while inducing him to produce. As every agent produces, there exists t^* agent that can be observed by each agent. Therefore proposed strategy is feasible.

Suppose (S, y) is a Nash network configuration where $\forall i \in N, y_i = 1$ but $T = \emptyset$. Then it must be that any $i \in N, 1 - \beta[|S_i|] - \gamma|S_i| < 0$ as otherwise it must be that $|S_i| \in T$. But then, no production and no observation $S'_i = \emptyset, y'_i = 0$ strictly dominates (S_i, y_i) contradicting that (S, y) is Nash. □

A non empty production inducement set does not guarantee that agent will always engage in production. When at least one agent must be observed to induce another to engage in production or when agents are indifferent between not producing or producing without observing any other agent it is possible that a society can be stuck in a stable, typically inefficient, state where no agent produces. This requirement, which is stated in our next result, simplifies to the condition that the cost of production, with no agent observed, is at least equal to the benefit from production.

Proposition 3. *Suppose $T \neq \emptyset$. Then a Nash network configuration (S, y) with no production exists if and only if $\beta[0] \geq 1$.*

Proof of Proposition 3. Suppose (S, y) is Nash network configuration satisfying $\forall i \in N, y_i = 0$ but $\beta[0] < 1$. Then for $i \in N$ the strategy $y'_i = 1, S'_i = \emptyset$ provides the payoff

[1] It is in fact easy to check that if $|\tilde{t}| = 2$ due to strict convexity of the cost function it must be that $t^* = \tilde{t}$.

$1 - \beta[0] > 0$ which is strictly higher than payoff from no production ($-|S_i|\gamma \leq 0$) contradicting with the assumption that (S, y) is a Nash network configuration.

Suppose (S, y) satisfies $\beta[0] \geq 1$ but (S, y) where $\forall i \in N, y_i = 0$ is not a Nash network configuration. Since no one else produces and links are costly, there is no benefit from observing, so the best response for typical agent i is $S_i = \emptyset$. If i produces its payoff is $1-\beta[0] < 0$ which is strictly less than the payoff of the strategy $y_i = 0$. $\square$

We now have a clear idea about the conditions under which we shall expect a society to engage in production completely or produce nothing at all. What naturally follows is under what conditions we shall expect societies with partial production, i.e. $\exists i, j \in N$ such that $y_i = 0, y_j = 1$. It turn outs that such situations require both producers and non producers to have identical payoffs. The reasoning behind this observation is the fact that both non producer and a producer can imitate each other's strategy, therefore shall not differentiate between the two. This in turn implies that producers gain no strict benefits from production. The condition for existence of such societies reduces to the condition that there shall be no strict benefits even when there are enough producers to observe to maximize the payoff. Our next result characterizes this possibility.

Proposition 4. *Suppose $T \neq \emptyset$. Then a Nash network configuration (S, y) with partial production exists if and only if $\forall j \in t^*, 1 - \beta[j] - \gamma j = 0$.*

Proof of Proposition 4. Let (S, y) be a Nash network configuration with partial production. Consider agents i and j with $y_i = 1, y_j = 0$. As $y_j = 0$ we can assume that $j \notin S_i$ and $S_j = \emptyset$. As both i and j can imitate each other's strategy, their payoffs must be same, i.e. $1-\beta[|S_i|]-\gamma(|S_i|) = 0$. As (y_i, S_i) is i's best response with 0 payoff, it must be that either $i.$ $|S_i| \in t^*$ or $ii.$ the marginal payoff from observing one more agent is still positive but such an agent does not exist, i.e. $\#\{l \in N | y_l = 1\} = |S_i|+1$.Case $ii.$ is not possible since agent j facing one more producer than agent i can set $S'_j = S_i \cup \{i\}, y'_i = 1$ and enjoy a strictly positive payoff. So it must be that $|S_i| \in t^*$, which yields $\tilde{t} = t^*$.

Suppose $\forall j \in t^*, 1-\beta[j]-\gamma j = 0$ which implies $\tilde{t} = t^*$. Define a partial production network as follows. Let $P = \{1, \ldots, l+1\}$ for $l \in \tilde{t}$. Set $\forall i \in P, y_i = 1, S_i = P \setminus \{i\}$ and $\forall j \in N \setminus P, y_j = 0, S_j = \emptyset$. It immediately follows from definition of set $\tilde{t}$ that above strategies for each type of agent is a best response, hence the network is a Nash configuration. $\square$

We shall now focus on the network structures that can emerge from the production configuration of a society. Our first result states that existence of complete network requires not only that each agent must produce but also that observing all other agents must be payoff maximizing, that is, $(n-1) \in t^*$.

Proposition 5. *A complete network is Nash if and only if $\forall i \in N, y_i = 1$ and $(n-1) \in t^*$.*

Proof of Proposition 5. Recall that a complete network is defined as $\forall i \in N, S_i = N \setminus \{i\}$, so $|S_i| = n - 1$.

If (S, y) is a Nash complete network, then $\gamma > 0$ implies that each link must be with a producer, so $\forall i \in N, y_i = 1$ and by definition of t^*, $(n-1) \in t^*$.

Similarly, if $\forall i \in N, y_i = 1, |S_i| = n - 1$ and $(n - 1) \in t^*$, for each agent $i \in N$, strategy (S_i, y_i) is best response by definition of t^*. $\square$

We already noted that while a non producing society must admit an empty network, it is still possible that a society partially or completely engages in production may admit an empty network. This requires either that it is optimal for agents not to observe even in the case of production, which can be due to high observation costs, or production costs must be prohibitively high to prevent agents from production. The following result states above observation formally.

Proposition 6. *There exists a Nash empty network (S, y) if and only if either $0 \in t^*$ or $\beta[0] \geq 1$.*

Proof of Proposition 6. Let (S, y) be an empty Nash network but $0 \notin t^*$ and $\beta[0] < 1$. As $\beta[0] < 1$, the strategy for typical agent i, $y_i = 1, S_i = \emptyset$ strictly dominates no production, thus $\forall i \in N, y_i = 1$. As $0 \in T$ we have $t^* \neq \emptyset$ and since $\forall i \in N, y_i = 1$, the strategy $y_i = 1, |S_i| \in t^*$ is a feasible best response strategy which will strictly dominate $y_i = 1, S_i = \emptyset$ (since $0 \notin t^*$ by assumption) contradicting with the assumption that (S, y) is an empty Nash network.

Suppose $0 \in t^*$, then for any $i \in N$, $y_i = 1, S_i = \emptyset$ is a feasible best response, so the empty network where $\forall i \in N, y_i = 0$ is Nash. (Note that if $\beta[0] = 1$ empty network configurations with partial and no production are also Nash).

Suppose $\beta[0] \geq 1$, then for any $i \in N$, $y_i = 0, S_i = \emptyset$ is a feasible best response, so the empty network where $\forall i \in N, y_i = 0$ is Nash. □

We shall now discuss, informally, when other well known network structures can emerge in our setting. Existence of links implies that there is complete or partial production in the society. In case of partial production, which we characterized in Proposition 4, links must be between producing agents. In the case of complete production, which we characterized in Proposition 2, it must be that optimal number of agents an agent wishes to observe must be between 0 (in which case empty network forms) and $n - 1$ (in which case complete network forms).

Consider the *wheel* network where each agent only observes a single agent forming a ring. This requires every agent to be producer, weakly benefiting from observing a single agent and no more. This implies necessary condition for existence of a wheel network is $1 \in t^*$.[2]

Consider the *star* network where $(n-1)$ agents observe a single central agent, and the central agent observes the $(n - 1)$ other agents. Again this implies a completely producing society. As every agent produces, any link strategy is feasible. If a single link and $(n - 1)$ links are chosen best responses it must be that $t^* = \{1, n - 1\}$. This, however, due to convexity of the cost function, is only possible if $1 = n-1$ or $n-1 = 2$, that is, either $n = 2$ or $n = 3$.[3]

We note that none of these structures can be supported as *strict* Nash network configurations.[4]

4 Efficient Network Configurations

We are interested in the network configurations that are efficient. As efficiency measure we consider the sum of payoffs of all agents in a society.[5]

$$W = \sum_{j \in N} \pi_j[S, y]$$

Our first result in this section characterizes the efficient network configurations. When production inducement set is empty, a society where no agent produces and no agent observes others is the unique efficient network configuration.

If there exists a number of producers that will provide strict benefits from production, efficiency requires all society to produce and each agent to observe optimal number of agents. When such strict benefits do not exist, but inducement set is not empty, efficient(societies consists of either (i) with no production, forming an empty network or (ii) with partial or complete production in which each producer i observes $|S_i| \in T = t^*$ agents.

Following proposition states this result.

[2]There are, in fact, three possibilities $t^* = \{1\}, \{0, 1\}$ or $t^* = \{1, 2\}$. It is also straightforward to show that this condition is also sufficient for existence of wheel networks.

[3]We can extend our result to other possible variations of a star network. For example if we define a star network as a single agent observed by all other members of a society only, then obviously the condition will be $t^* = \{0, 1\}$. If we consider star networks where a single agent observes all others in a society, then the condition will be $t^* = \{0, n - 1\}$.

[4]A strict Nash equilibrium is a Nash equilibrium where for each player the strategy is the unique best response.

[5]EEE Mention other measures that are possible and why we choose this. EEE

Proposition 7. (i) *If* $T = \emptyset$*, empty network with no production is the unique efficient network configuration.* (ii) *If* $\exists i \in T$ *such that* $1 - \beta[i] - i\gamma > 0$*, then* (S, y) *satisfying* $\forall i \in N, y_i = 1, |S_i| \in t^*$ *is the unique efficient network configuration.* (iii) *If* $\forall i \in T, 1 - \beta[i] - i\gamma = 0$*, then, both the empty network with no production , and any network configuration where each producer connects* $i \in T$ *other producers is efficient.*

Proof of Proposition 7 . (i) As $T = \emptyset$, the configuration $S_i = \emptyset, y_i = 0$ is the unique payoff maximizing strategy for all $i \in N$. Therefore, the payoff of each agent in the empty network with no production strictly dominates corresponding agent's payoff in other configurations.

(ii) By definition $\forall i \in N, y_i = 1, |S_i| \in t^*$ provides the unique highest payoff provided that $t^* \neq \emptyset$ and there are at least $|S_i|$ producers other than i. As $\forall i \in N, y_i = 1$ and $T \neq \emptyset$ both conditions are satisfied.

(iii) Let (S, y) be an efficient network. If $y_i = 1$ for some $i \in N$, and there are at least $k \in T$ producers, by definition linking i with $k \in T = t^*$ other producers maximizes i's payoff, setting it $\pi_i[S, y] = 0$, thus increases total welfare. Otherwise, linking with $k' \notin T$ producers, by definition will provide strictly less payoff, i.e. $\pi_i[S, y] < 0$, and the strategy $S'_i = \emptyset, y'_i = 0$ providing payoff 0 strictly dominates. The result directly follows from these observations. □

Proposition 8. *Every efficient network configuration is stable.*

Proof. When $T = \emptyset$, by Proposition 7 empty network with no production is the unique efficient network as well as due to Proposition 1 is also stable. When there exist $i \in T$ such that $1 - \beta[i] - i\gamma > 0$, by Proposition 7 network configuration (S, y) where $\forall i \in N, y_i = 1, |S_i| \in t^*$ is efficient. By Proposition 2 given $T \neq \emptyset$ there exists a Nash network configuration where $\forall i \in N, y_i = 1$. As by definition linking with $|S_i| \in t^*$ agents is best response the efficient network (S, y) is also stable. When $\forall i \in T, 1 - \beta[i] - i\gamma = 0$, by Proposition 1 both the empty network with no production, and any network configuration where each producer connects $i \in T$ other producers is efficient. Note that the condition implies $T = \tilde{t} = t^*$. Then by Proposition 4 there always exists a partial production network where each agent connects $i \in t^* = T$ agents. Note that the condition $\forall i \in T, 1 - \beta[i] - i\gamma = 0$ implies $\beta[0] \geq 1$, which due to Proposition 6, yields that empty network configuration with no production is also stable. □

But it is clear that not every Nash network is efficient. For this consider the following society.

5 An Example

As an illustrative example we assume a cost function of the form $\beta[x] = \bar{\beta} e^{-\alpha x}$, where $\alpha > 0$ which yields $\underline{\beta} = 0$ and $\beta[0] = \bar{\beta}$.

Consider the complete network. Proposition 5 states that $(n-1) \in t^*$ is necessary and sufficient condition for existence of complete network. We first construct the condition $(n-1) \in T$

$$1 - \bar{\beta} e^{-(n-1)} - \gamma(n-1) \geq 0 \Rightarrow \beta \leq \frac{1 - \gamma(n-1)}{e^{-\alpha(n-1)}}$$

To guarantee that $(n-1)$ also belongs to set t^*, due to the strict convexity of the cost function, it is sufficient to set the parameters so that connecting with $(n-2)$ agent provides (weakly) less payoff then connecting $(n-1)$ agents

$$1 - \bar{\beta} e^{-\alpha(n-1)} - \gamma(n-1) \geq 1 - \bar{\beta} e^{-\alpha(n-2)} - \gamma(n-2)$$

which yields the condition

$$\bar{\beta} \geq \frac{\gamma}{e^{-\alpha(n-2)} - e^{-\alpha(n-1)}}$$

Thus, the interval for stable complete network is

$$\frac{1-\gamma(n-1)}{e^{-\alpha(n-1)}} \geq \bar{\beta} \geq \frac{\gamma}{e^{-\alpha(n-2)} - e^{-\alpha(n-1)}}$$

which is the triangle indicated with the doted lines in figure 1.

Now consider the empty network which, due to Proposition 6, requires (i) $\bar{\beta} \geq 1$ or (ii) $0 \in t^*$. The interval for condition (ii) can calculated by checking first that $0 \in T$ and then $0 \in t^*$ which show below

$$1 - \bar{\beta} \geq 0 \Rightarrow \bar{\beta} \leq 1$$

$$1 - \bar{\beta} \geq 1 - \bar{\beta}e^{-1} - \gamma \Rightarrow \bar{\beta} \leq \frac{\gamma}{1 - 1/e}$$

When (i) is satisfied with strict inequality, no agent produces unless they observe some other producer. So stable empty network configurations cannot involve any production. When (ii) is satisfied with strict inequalities, then stable empty network configurations involve full production. Note that both (i) and (ii) can simultaneously be satisfied only when $\bar{\beta} = 1$ and $\bar{\beta} \leq \frac{\gamma}{1-1/e}$, in which case we can have stable empty network configurations than can have partial production.

Figure 1 shows the stability intervals for complete and empty networks with different production configurations.

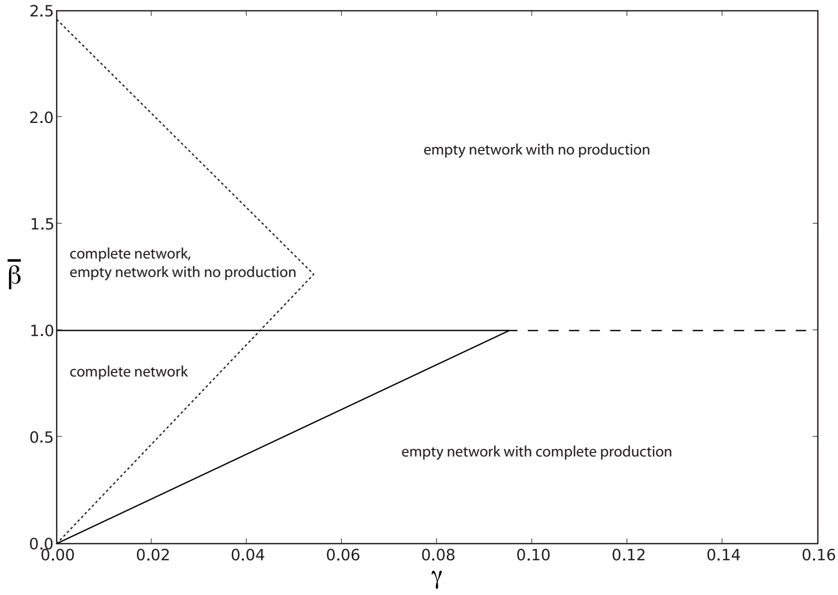

Figure 1: Stable Empty and Complete Networks, $n = 10, \alpha = 0.1$.

Figure 2 shows the intervals for stable network configurations according to their production configurations. When the production inducement set is empty, the unique stable network configuration empty network with no production (Proposition 1). When this set is non empty different stable configurations are possible.

6 Conclusion

We provide characterization of both the production decisions of agents in a society and the network implied by their observation behavior.

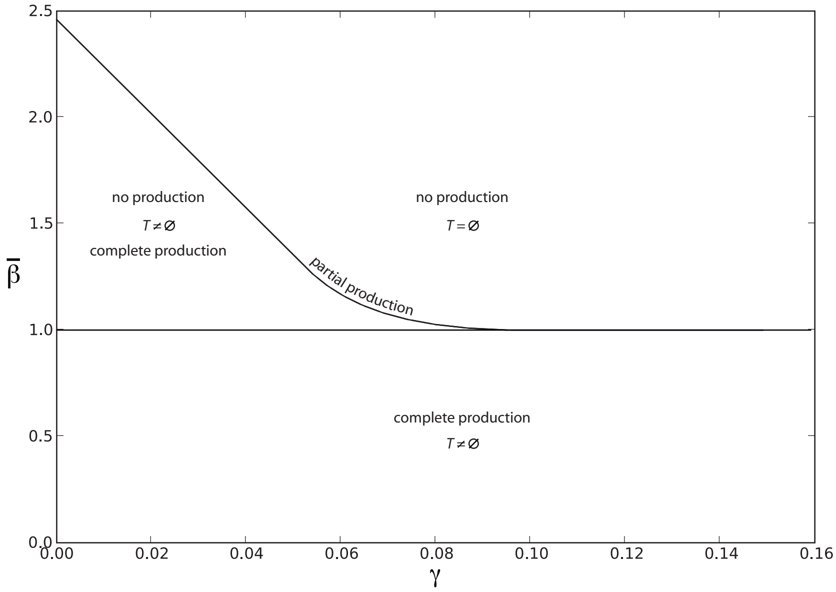

Figure 2: Stable Networks, $n = 10, \alpha = 0.1$.

We illustrate our results with an example. Let us summarize our results by using the example given in section 5. When initial cost of production is sufficiently low ($\bar{\beta} < 1$) every agent strictly prefers to produce even without observing others, therefore all stable configurations must involve full production. Benefits from observing others will depend on the link cost γ. When γ is sufficiently low, every agent observes others, further reducing their production costs. When γ is high, agents stop observing others, yet still continue production albeit with higher production cost $\bar{\beta}$. In the intermediate cost range for γ, shown as the empty triangle in figure 1, all agents engage in production and strictly prefer to observe others, yet observe agents in the range $(n-1)$ and 0. In this region network structures other than empty and complete, e.g. wheel network, are stable with complete production.

When initial cost of production is high ($\bar{\beta} < 1$) production requires coordination, that is, no agent will engage in production unless there exists other producers in the society. When the observing cost γ is prohibitively high, that is, cost benefits from observing others cannot justify the cost of observing, no one observes and no one produces. If costs are sufficiently low, every body is willing to observe others and produce, thus forming the complete network configuration.

Note that these are not necessarily the unique network configurations. Consider the tip of the triangle indicating the stable interval for the complete network. Why does the complete network becomes unstable if $\bar{\beta}$ decreases? In this range there are other stable network configurations that can coexists with empty network configuration. The reason that complete network becomes unstable is that the optimal number of agents to observe decreases, since production cost is already lower, thus agents will prefer to observe less than $(n-1)$ agents, but will still produce as long as there are sufficient producers to observe.

The model we provide here forms the basis for a rich set of extensions. One possibility of extension is to consider asymmetries in payoffs, due to say cost heterogeneity. In such a setting we expect agents with cost benefits to have production inducement sets that to be super set of inducement sets of agents with cost disadvantage.

A shortcoming of our model is that since agents are symmetric, one does not differentiate between observing any two agents as long as both engage in production.

So network structures are not too informative. What naturally follows is introducing heterogenity to the society.

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