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Derivation Notes 02
MRT=MC$_X$/MC$_Y$

Suppose that production technologies are $X = F(K_X, L_X)$ and $Y = G(K_Y, L_Y)$ and resource constraints are defined as $\overline{K} = K_X + K_Y$ and $\overline{L} = L_X + L_Y$. Show that MRT= MC$_X$/MC$_Y$.

First, totally differentiate X and Y:

$$dX = F_{K_X} dK_X + F_{L_X} dL_X \tag{1a}$$
$$dY = G_{K_Y} dK_Y + G_{L_Y} dL_Y \tag{1b}$$

Secondly, totally differentiate the resource constraints:

$$d\overline{K} = 0 = dK_X + dK_Y \tag{2a}$$
$$d\overline{L} = 0 = dL_X + dL_Y \tag{2b}$$

Equation (2) implies that

$$dK_Y = -dK_X \tag{3a}$$
$$dL_Y = -dL_X \tag{3b}$$

From (1):

$$\frac{dY}{dX} = \frac{G_{K_Y} dK_Y + G_{L_Y} dL_Y}{F_{K_X} dK_X + F_{L_X} dL_X} \tag{4}$$

Using (3) in (4), we get:

$$\frac{dY}{dX} = \frac{G_{K_Y}(-dK_X) + G_{L_Y}(-dL_X)}{F_{K_X} dK_X + F_{L_X} dL_X} \tag{5}$$

We may re-write (5) as follows:

$$-\frac{dY}{dX} = \frac{G_{K_Y}\{(G_{L_Y}/G_{K_Y})dL_X + dK_X\}}{F_{K_X}\{(F_{L_X}/F_{K_X})dL_X + dK_X\}} \tag{6}$$

From production efficiency, we know that $G_{L_Y} / G_{K_Y} = F_{L_X} / F_{K_X}$.[1] Hence,

$$-\frac{dY}{dX} = \frac{G_{K_Y}}{F_{K_X}} \tag{6}$$

Next, from the profit maximization condition, you know that

$$P_X F_{K_X} = r \tag{7a}$$

$$P_Y G_{K_Y} = r \tag{7b}$$

You also know that

$$P_X = MC_X \tag{8a}$$

$$P_Y = MC_Y \tag{8b}$$

Hence, using (7) and (8) in (6),

$$-\frac{dY}{dX} = \frac{r / P_Y}{r / P_X} = \frac{P_X}{P_Y} = \frac{MC_X}{MC_Y} \tag{9}$$

Hence,

$$MRT = \frac{MC_X}{MC_Y} \tag{10}$$

[1] Recall that production efficiency imposes $\left(\frac{MPP_L}{MPP_K}\right)_X = \left(\frac{MPP_L}{MPP_K}\right)_Y$