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Derivation Notes 01
Production Possibility Frontier

Example 1

Suppose that

$$Y_1 = L_1^{\alpha} \qquad Y_2 = L_2^{\beta} \tag{1}$$

$$L_1 + L_2 = L \tag{2}$$

From production technology, we can re-write (1) and (2) as follows:

$$Y_1^{1/\alpha} = L_1 \qquad Y_2^{1/\beta} = L_2 \tag{3}$$

Next, from the resource constraint equation, it is easy to see that

$$L = Y_1^{1/\alpha} + Y_2^{1/\beta} \tag{4}$$

Note that $\partial Y_2 / \partial Y_1 < 0$ and $\partial^2 Y_2 / \partial Y_1^2 < 0$ (simply follow implicit differentiation rule).

Example 2

Suppose that

$$Y_1 = K_1^{\alpha} L_1^{1-\alpha} \qquad Y_2 = K_2^{\beta} L_2^{1-\beta} \tag{1}$$

$$L_1 + L_2 = L \qquad K_1 + K_2 = K \tag{2}$$

From profit maximization, it is obvious that

$$\frac{\partial Y_1 / \partial K_1}{\partial Y_1 / \partial L_1} = \frac{\partial Y_2 / \partial K_2}{\partial Y_2 / \partial L_2} \tag{3}$$

Case #1: Suppose that $\alpha = \beta$. Then we can derive the PPF in an explicit form. First, from equation (3) it easy to see that

$$\frac{K_1}{L_1} = \frac{K_2}{L_2} \tag{4}$$

Then use equation (2) in equation (4) and get

$$\frac{K_1}{L_1} = \frac{K - K_1}{L - L_1} \Rightarrow$$

$$K_1 = K\frac{L_1}{L} \tag{5}$$

Now, beginning from production functions with $\alpha = \beta$, do the followings:

$$Y_1 = \left(K\frac{L_1}{L}\right)^{\alpha} L_1^{1-\alpha} \qquad Y_2 = (K - K_1)^{\alpha}(L - L_1)^{1-\alpha}$$

$$Y_1 = \left(\frac{K}{L}\right)^{\alpha} L_1 \qquad Y_2 = \left(\frac{K}{L}\right)^{\alpha}(L - L_1)$$

$$L_1 = \left(\frac{L}{K}\right)^{\alpha} Y_1 \qquad L_1 = L - Y_2\left(\frac{L}{K}\right)^{\alpha} \tag{6}$$

Using the fact that $L_1 \equiv L_1$ in (6), one gets

$$Y_1 + Y_2 = K^{\alpha}L^{1-\alpha} \tag{7}$$

which is the PPF in case of $\alpha = \beta$. Note that slope of the PPF is –1 and its second derivative is zero.

Case #2: $\alpha \neq \beta$,

In that case it is *impossible* to derive the PPF in an explicit form. The corresponding solution of equation (6) in case #2 is

$$Y_1 = K_1^{\alpha}\left(\frac{aK_1L}{K + (a-1)K_1}\right)^{1-\alpha} \qquad Y_2 = (K - K_1)^{\beta}\left(L - \frac{aK_1L}{K + (a-1)K_1}\right)^{1-\beta}$$

where $a = \left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$.