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ECON 202
INTERMEDIATE MACROECONOMICS **26 April 2007**
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Key to Midterm Exam

1. (**15 Points**) Calculate the GDP of Farmland, a fictitious economy whose numbers are listed below. Do so using all three methods (value added approach, income approach, and expenditure approach).

Farmland, year 2000

Farmer Jones, (private firm)	
Corn Sold to Govt	$35
Corn Sold to Singapore	$25
Corn Sold to FoodCo, Inc	$20
Paid workers	$40
Tax on profit	$15

FoodCo, Inc	
Sold Corn Flakes to Consumers	$100
Farmland sales tax	$10
Revenue of FoodCo, Inc	$90
Bought corn from Farmer Jones	$20
Corn Inventory	
Beginning of Year	$0
End of Year	$10
Bought salt from Egypt	$10
Paid workers	$20
Tax on Profit	$15

Farmland Govt	
Taxes	$50
Purchase of Corn	$35
Unemployment benefits Paid	$15

Households	
Taxes on wage income	$10
Unemployment benefits	$15

Value-added Approach: 80 + [100-10-10] = 160 **5 points**
Expenditure Approach: 100 + 10 + 35 + (25-15) = 160 **5 points**
Income Approach: 60 + 60 + 40 = 160 **5 points**
W Π TA

Grading: 5 points each

2. (**10 Points**) Fill in the blanks in the following table:

Year	Nominal GDP	Real GDP	GDP Deflator (1994=100)	Real GDP Growth Rate
1992	**5931**	**6244**	**95**	**-**
1993	**6240**	**6389**	**97.6**	**2.32%**
1994	**6611**	**6611**	**100**	**3.47%**
1995	**7234**	**6761**	**107**	**2.26%**
1996	**7290**	**6994**	**104**	**3.44%**

Grading: 1 point each

3. (**15 Points**) Suppose that utility function u of a representative agent is $u = c^{0.75} l^{0.25}$, where c is consumption of physical goods and l is consumption of leisure. Assume that non-labor income is 120, the real wage rate is $w = 5$ and $h = 24$ hours. Find the optimal values of c, l, N, and u.

The Lagrange is: $L = C^{0.75} l^{0.25} - \lambda\{C + 5l - 120 - 120\}$. The first order conditions are:

$$\frac{\partial L}{\partial C} = 0 \Rightarrow (0.75)C^{-0.25} l^{0.25} - \lambda = 0 \qquad \text{(Equation 1)}$$

$$\frac{\partial L}{\partial l} = 0 \Rightarrow (0.25)C^{0.75} l^{-0.75} - \lambda 5 = 0 \qquad \text{(Equation 2)} \qquad \text{(3 points)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow C + 5l - 120 - 120 = 0 \qquad \text{(Equation 3)}$$

From the first two first-order conditions (i.e., from (1) and (2)), we obtain:

$$\frac{(0.75)C^{-0.25} l^{0.25}}{(0.25)C^{0.75} l^{-0.75}} = \frac{\lambda}{5\lambda} \Rightarrow C = 15l \qquad \text{(2 points)}$$

Using this result in the third first-order condition:

$$15l + 5l = 240 \Rightarrow 20l = 240 \Rightarrow l^* = 12 \qquad \text{(2 points)}$$

Since $N + l = 24$, $N^* = 12$. (2 points)

Optimal consumption can be found by using for example $C = 15l$ (the budget constraint can also be used):

$$C^* = 15l^* \Rightarrow C^* = 15(12) \Rightarrow C^* = 180 \qquad \text{(2 points)}$$

Finally, utility (=happiness) can be found as:

$$u^* = (C^*)^{0.75} (l^*)^{0.25} \Rightarrow u^* = (180)^{0.75} (12)^{0.25} = 91.4 \qquad \text{(4 points)}$$

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4. (**30 Points**) Suppose that utility function u of a representative agent is $u = c^{0.25} l^{0.75}$, where c is consumption of physical goods and l is consumption of leisure. Suppose that production technology is represented by $y = 2K^{0.35} N^{0.65}$ where $K = 48$ is the physical capital stock and N is labor. We assume that $h = 24$, $h = l + N$ and that there is no government in the economy (use w and π to denote the real wage and profits, respectively)..

a) Find the optimal values of c, l, N, y, w, π, and u under the competitive equilibrium assumption.

b) Find the optimal values of c, l, N, y, and u under the social planner's solution assumption. Are the results different or same? Why or why not?

This is a General Equilibrium Model. Let us start from the household's problem.
The Lagrange is: $L = \mathrm{C}^{0.25} l^{0.75} - \lambda\{C + wl - wh - \pi\}$. The first order conditions are:

$$\frac{\partial L}{\partial C} = 0 \Rightarrow (0.25)\mathrm{C}^{-0.75} l^{0.75} - \lambda = 0 \qquad \text{(Equation 1)}$$

$$\frac{\partial L}{\partial l} = 0 \Rightarrow (0.75)\mathrm{C}^{0.25} l^{-0.25} - \lambda w = 0 \qquad \text{(Equation 2)} \qquad \text{(3 points)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow C + wl - wh - \pi = 0 \qquad \text{(Equation 3)}$$

From the first two first-order conditions (i.e., from (1) and (2)), we obtain:

$$\frac{(0.25)\mathrm{C}^{-0.75} l^{0.75}}{(0.75)\mathrm{C}^{0.25} l^{-0.25}} = \frac{\lambda}{w\lambda} \Rightarrow C = \frac{wl}{3}$$

Using this result in the third first-order condition:

$$\frac{wl}{3} + wl = wh + \pi \Rightarrow \frac{4wl}{3} = wh + \pi \Rightarrow l = \frac{3}{4}h + \frac{3}{4}\frac{\pi}{w} \Rightarrow l = 18 + \frac{3}{4}\frac{\pi}{w}.$$

In order to solve the model, we need labor supply, which may be obtained directly from $N + l = 24$:

$$N^s = 24 - l \Rightarrow N^s = 24 - \left(18 + \frac{3}{4}\frac{\pi}{w}\right) \Rightarrow N^s = 6 - \frac{3}{4}\frac{\pi}{w}.$$

We cannot solve the problem unless π is determined. For this, let us look at the production side. From the firm's profit maximization problem:

$$\pi = 2K^{0.35}N^{0.65} - wN \Rightarrow \frac{d\pi}{dN} = 2(0.65)K^{0.35}N^{-0.35} - w = 0 \Rightarrow N^d = \left(\frac{1.3}{w}\right)^{\frac{1}{0.35}} K.$$

We need to also calculate profits:

$$\pi = (2K^{0.35}N^{-0.35} - w)N \Rightarrow \pi = (2K^{0.35}\frac{w}{2(0.65)K^{0.35}} - w)N \Rightarrow \pi = (0.5384)wN.$$

Now we have enough information to solve the general equilibrium. First, use the profit equation in the labor supply:

$$N^s = 6 - (0.75)\frac{(0.5384)wN}{w} \Rightarrow N^s = 6 - (0.75)(0.5384)N \Rightarrow N^s = 6 - (0.4038)N$$

Given that $N^s = N^d$ at equilibrium,

$$N = 6 - (0.4038)N \Rightarrow (1.4038)N = 6 \Rightarrow N^* = 4.27 \qquad \text{(2 points)}$$

Using this information at $N^d = \left(\frac{1.3}{w}\right)^{\frac{1}{0.35}} K$ implies $w^* = 3.0309$. (2 points)

The rest can be calculated by substitution:

$\pi^* = 6.968$ (2 points)
$l^* = 19.73$ (2 points)
$C^* = 19.93$ (2 points)
$y^* = 19.93$ (2 points)
$u^* = 19.97$ (3 points)

Social Planner's Problem:

For this approach, we setup the following problem:

The Lagrange is: $L = C^{0.25} l^{0.75} - \lambda\{C - 2K^{0.35} N^{0.65}\}$. The first order conditions are:

$$\frac{\partial L}{\partial C} = 0 \Rightarrow (0.25)C^{-0.75} l^{0.75} - \lambda = 0 \qquad \text{(Equation 1)}$$

$$\frac{\partial L}{\partial l} = 0 \Rightarrow (0.75)C^{0.25} l^{-0.25} - \lambda\{-2(0.65)K^{0.35}(h-l)^{-0.35}(-1)\} = 0 \qquad \text{(Equation 2)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow C = 2K^{0.35} N^{0.65} \qquad \text{(Equation 3)}$$

(Deriving the first-order conditions is 3 points)

From the first two first-order conditions (i.e., from (1) and (2)), we obtain:

$$\frac{(0.25)C^{-0.75} l^{0.75}}{(0.75)C^{0.25} l^{-0.25}} = \frac{1}{(5.039)(24-l)^{-0.35}} \Rightarrow C = (1.679)(24-l)^{-0.35} l$$

Using this result for C in the third first-order condition:

$$(1.679)(24-l)^{-0.35} l = (7.75)(24-l)^{0.65} \Rightarrow l^* = 19.73 \qquad \text{(2 points)}$$

The rest follows from substitution:

$N^* = 4.27$ (2 points)

$C^* = 19.93$ (2 points)

$y^* = 19.93$ (2 points)

$u^* = 19.97$ (3 points)

Note that we do not have w^* and π^* in the social planner's problem.

5. (**10 Points**) Suppose that Daniel has income of y_1 when he is young and y_2 when he is old. The real interest rate is $r=1$. The overall utility function of Daniel is $U=2c_1^{0.5}+(0.5)2c_2^{0.5}$.

(i) Find the optimal values of c_1, c_2 and s.

(ii) Show that $\frac{\partial s}{\partial y_1}>0$ and $\frac{\partial s}{\partial y_2}<0$. Interpret these results.

This is a Partial Equilibrium Model. The household's problem is: $L=2c_1^{0.5}+(0.5)2c_2^{0.5}-\lambda\left\{c_1+\frac{c_2}{2}-y_1-\frac{y_2}{2}\right\}$. The first order conditions are:

$$\frac{\partial L}{\partial c_1}=0\Rightarrow \mathrm{c}_1^{-0.5}-\lambda=0 \qquad \text{(Equation 1)}$$

$$\frac{\partial L}{\partial c_2}=0\Rightarrow (0.5)\mathrm{c}_2^{-0.5}-\frac{\lambda}{2}=0 \qquad \text{(Equation 2)} \qquad \text{(3 points)}$$

$$\frac{\partial L}{\partial \lambda}=0\Rightarrow c_1+\frac{c_2}{2}-y_1-\frac{y_2}{2}=0 \qquad \text{(Equation 3)}$$

From the first two first-order conditions (i.e., from (1) and (2)), we obtain:

$$\frac{\mathrm{c}_1^{-0.5}}{(0.5)\mathrm{c}_2^{-0.5}}=\frac{\lambda}{\frac{\lambda}{2}}\Rightarrow c_1=c_2$$

Using this result in the third first-order condition: $c_2+\frac{c_2}{2}=y_1+\frac{y_2}{2}\Rightarrow$

$$c_1^*=\frac{2}{3}\left(y_1+\frac{y_2}{2}\right) \qquad \text{(2 points)}$$

$$c_2^*=\frac{2}{3}\left(y_1+\frac{y_2}{2}\right) \qquad \text{(2 points)}$$

$$s^*=\frac{1}{3}(y_1-y_2) \qquad \text{(2 points)}$$

It is obvious that

$$\frac{\partial s^*}{\partial y_1}=\frac{1}{3}>0 \qquad \text{(2 points)}$$

$$\frac{\partial s^*}{\partial y_2}=-\frac{1}{3}<0 \qquad \text{(2 points)}$$

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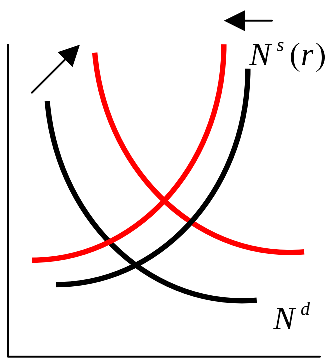
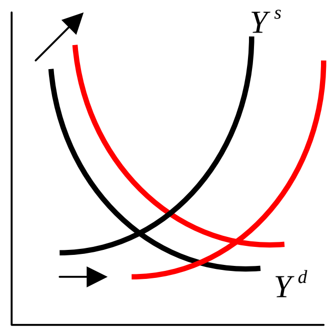
6. (**15 Points**) An unexpected invention makes supersonic transportation substantially cheaper. This invention is expected to raise overall productivity both in the current period and in the future period. What will happen to current values of **N**, **I**, **C**, **Y**, **w**, and **r**? UU**Discuss and illustrate**.



(3 points)



(3 points)

$z_1 \uparrow \Rightarrow N^d \uparrow^+ \Rightarrow Y^s \uparrow^+ \Rightarrow (r \downarrow, Y \uparrow)$ (3 points)

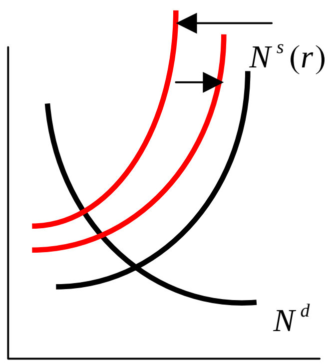
$z_2 \uparrow \Rightarrow I^d \uparrow^+ \Rightarrow Y^d \uparrow^+ \Rightarrow (r \uparrow, Y \uparrow)$ (3 points)

It is more likely that $(r \downarrow, Y \uparrow)$. If so, then $N^s \uparrow^-$.

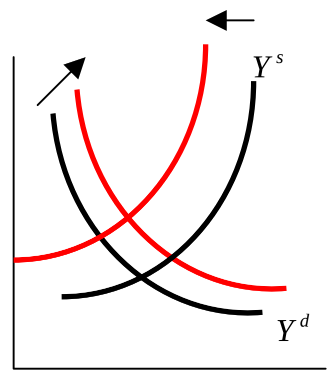
In sum, when $(z_1 \uparrow, z_2 \uparrow)$ ➔ $C \uparrow$, $I \uparrow$, $N \uparrow$, $w \uparrow$. (3 points)

(With some explanation)

7. (**10 Points**) The government announces that a decrease in government expenditure will occur next year. What effect will this have on the current values of aggregate output, employment, real wage, the real interest rate, consumption, and investment? **Discuss and illustrate**.



(3 points)



(3 points)

$G_2 \uparrow \Rightarrow we \uparrow \Rightarrow N^s \uparrow^- \Rightarrow (r \uparrow, Y \downarrow)$

$G_2 \uparrow \Rightarrow C^d \uparrow^+ \Rightarrow Y^d \uparrow^+ \Rightarrow (r \uparrow, Y \uparrow)$ (3 points)

$(r \uparrow, Y ?)$. Since $(r \uparrow)$, then $N^s \uparrow^+$.

In sum, when $(G_2 \uparrow)$ ➔ $C \uparrow$, $I \downarrow$, $N \uparrow$, $Y \uparrow\downarrow ?$. (1 point)
(With some explanation)