

**ECON 202**  
**INTERMEDIATE MACROECONOMICS**  
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**Key to Exercise VII**  
**Multiplier Analysis**

1. A macroeconomy is described by the following relationships:

Consumption:  $C = \bar{C} + cYD$

Investment:  $\bar{I}$

Government expenditure:  $\bar{G}$

Income Tax Revenue:  $TA = \bar{t}Y$

where  $\bar{C}$ ,  $c$ ,  $\bar{I}$ ,  $\bar{G}$ , and  $\bar{t}$  are constant values,  $Y$  is total income and  $YD$  is disposable income.

a. Find the equilibrium output for this economy

$$Y=C+I+G \rightarrow Y = \bar{C} + c(Y - tY) + \bar{G} + \bar{I} \rightarrow Y = \frac{\bar{A}}{1 - c(1 - t)}, \text{ where } \bar{A} = \bar{C} + \bar{G} + \bar{I}$$

b. Indicate the multiplier for this economy. Does charging a tax increase or decrease the multiplier?

$$\alpha = \frac{1}{1 - c(1 - t)}. \text{ The tax rate decreases the multiplier.}$$

c. Find the budget surplus ( $BS = TA - G$ ).

$$BS=TA-G \rightarrow BS=tY-G \rightarrow BS = t \frac{\bar{A}}{1 - c(1 - t)} - \bar{G}.$$

d. Suppose the parameters take on the following values:

$$\bar{C} = 40; c = 0.8; \bar{I} = 35; \bar{G} = 15; \bar{t} = 0.2$$

Use these formulae to find numerical values for equilibrium output, the multiplier, and the budget surplus.

First, note that  $\bar{A} = \bar{C} + \bar{G} + \bar{I}$  is  $\bar{A} = 90$ .

$Y = 90 / (0.36) \rightarrow Y = 250$ ;  $\alpha = 2.78$ ; and  $BS = (0.2)250 - 15 = 35$ .

2. Consider the following model of the economy:

$$C = 50 + 0.6(Y - TA + TR); I = 100; G = 100; X = M = 0;$$

where C is consumption, Y is income, TA is taxes, TR is transfers, I is investment, G is government spending, X is exports and M is imports.

a. State the equilibrium condition for GDP (national income) and solve for equilibrium national income for  $TA = 100$  (lump-sum tax) and  $TR = 0$ .

$$\begin{aligned} YD = Y - TA = Y - 100; C = 50 + (0.6)(Y - 100); &\rightarrow Y = 50 + (0.6)(Y - 100) + 100 + 100 \\ &\rightarrow Y = 190 + (0.6)Y \\ &\rightarrow Y = 475 \end{aligned}$$

b. The government decides to increase spending by 10. If it doesn't raise taxes, what will the new values of autonomous spending, the multiplier, and equilibrium income be? If the government raises taxes at the same time to maintain a balanced budget, what will the new values of autonomous spending, the multiplier, and equilibrium income be? Give a brief explanation of why income changed by as much or as little as it did.

The new value of Autonomous Spending A will be  $190 + 10 = 200$ . The multiplier does not change, that is, it is still 2.5. Equilibrium income will rise to 500.

If TA also increases by 10, then the new values of autonomous spending will be 194. The multiplier does not change. And equilibrium income becomes 485. Below, we show calculations.

$$\begin{aligned} YD = Y - TA = Y - 110; C = 50 + (0.6)(Y - 110); &\rightarrow Y = 50 + (0.6)(Y - 110) + 110 + 100 \\ &\rightarrow Y = 194 + (0.6)Y \\ &\rightarrow Y = 485 \end{aligned}$$

Income increases less when a balanced budget strategy is followed because then the increase in TA lowers disposable income available for further consumption.

c. Now, instead of assuming that the government collects a fixed amount of taxes, assume that it collects a fixed percentage of national income:  $T = tY$ . Assuming the tax rate,  $t$ , is 0.1667, solve for equilibrium income, autonomous spending, and the multiplier. Explain any differences with your answers to part b. Is the government budget balanced? What happens now if the government increases spending by 10? State and briefly explain the changes in equilibrium income and the government budget deficit.

$$\begin{aligned} YD = Y - TA = Y - tY = (1-t)Y; C = 50 + (0.6)(1-t)Y; &\rightarrow Y = 50 + (0.6)(1-t)Y + 100 + 100 \\ &\rightarrow Y = 250 + (0.5)Y \end{aligned}$$

$$\rightarrow Y=500$$

Hence, equilibrium income is  $Y=500$ , Autonomous spending is  $A=250$  and the multiplier is 2. Note that the equilibrium income is higher though the multiplier is smaller.

The Budget Surplus is  $BS=(0.167)(500)-100=83-100=-17$ .

If the government expenditure increased by 10, then the new equilibrium income would be

$$\rightarrow Y=260+(0.5)Y$$

$$\rightarrow Y=520$$

Alternatively, given multiplier 2,  $\Delta Y=2*10=20 \rightarrow 500+20=520$

The Budget Surplus becomes  $BS=(0.167)(520)-110=87-110=-23$ . The deficit increases.

3. Consider the following Goods Market model in a closed economy:

$$C = \bar{C} + cYD \quad (1)$$

$$I = \bar{I} - b \cdot r \quad (2)$$

$$G = \bar{G} \quad (3)$$

$$TA = tY \quad (4)$$

where  $Y$  is GDP,  $C$  is consumption,  $G$  is government spending,  $YD$  is disposal income,  $r$  is the interest rate,  $\bar{I}$  is autonomous investment,  $I$  is realized (actual) investment,  $t$  is the tax rate, and  $TA$  is tax revenue.

(i) Interpret the parameters in the equations above.

In equation (1),  $c$  is the *marginal propensity to consume* out of disposable income.  $mpc$  indicates the portion of disposable income that is allocated for consumption (the rest goes to saving). Parameter  $b$  is the marginal responsiveness of investment to interest rate. A one percentage decline (an increase) in the interest rate increases (decreases) investment by  $b$  percentage points (because the interest rate is in percentage points).  $t$  is the tax rate and total tax revenue is proportional to income. The higher the  $t$ , the higher the tax revenues,  $TA$ , of the government.

(ii) What are the *endogenous* and *exogenous* variables in the above model?

Exogenous variables in the model are  $\bar{C}$ ,  $\bar{I}$ , and  $\bar{G}$ .  $C$ ,  $YD$ ,  $I$ ,  $Y$ ,  $TA$ , and  $r$  are endogenous variables of the model (as we will also introduce the money market!).

(iii) Solve for the equilibrium output in the goods market. Represent equilibrium in the AD-Y space.

$Y=AD \rightarrow Y = \bar{C} + cYD + \bar{I} - b \cdot r + \bar{G} \rightarrow Y = \bar{C} + c(Y - tY) + \bar{I} - b \cdot r + \bar{G} \rightarrow$   
 $Y = \bar{C} + c(1-t)Y + \bar{I} - b \cdot r + \bar{G} \rightarrow Y = \bar{A} + c(1-t)Y - b \cdot r$  where  $\bar{A} = \bar{C} + \bar{I} + \bar{G}$ . One can draw Y=AD equilibrium given  $r$ .

(iv) Plot and show the slope of the IS curve in the  $r$ - $Y$  space.

Straightforward.

(v) What is the effect on output and consumption of a fiscal expansion financed by a similar increase in tax revenues:  $\Delta G = \Delta TA$ . Does the budget deficit  $BD = G - TA$ , change in any particular way?

We know that  $\Delta Y = c\Delta Y - c\Delta TA + \Delta G \rightarrow \Delta Y = \Delta G$ . In that case, consumption would not change ( $\Delta C = c(\Delta Y - \Delta TA) = c(\Delta Y - \Delta G) = 0$ ). BS will remain same as the assumption  $\Delta G = \Delta TA$  implies  $\Delta BS = 0$ .

Now assume that the money demand and supply are the following (here  $P=1$ ):

$$L = kY - h \cdot r \quad (5)$$

$$M^s = M \quad (6)$$

(vi) What is the equilibrium condition? Represent equilibrium in the  $r$ - $M$  space.

Equilibrium condition is simply  $L=M$

(vii) Interpret the parameters in equation (5).

The parameter  $k$  is the responsiveness of money demand for income changes (transactions motive) and the parameter  $h$  is the responsiveness of money demand for changes in the interest rate.

(viii) Plot and show the slope of the LM curve in the  $i$ - $Y$  space.

Straightforward.

(ix) How does the slope depend on  $k$ ? On  $h$ ? Give an economic interpretation.

The higher the parameter  $k$  and the smaller the parameter  $h$ , the flatter is the LM curve.

(x) Solve for both the LM and IS equation and find the equilibrium output.

Straightforward.

4. Answer the following questions based on the information below:

$$\begin{aligned}C &= 500 + .9(Y-TA) \\M_d &= 3000 + .1Y - 5000 r \\M_s &= 4000 \\TA &= 500 + .1(Y) \\IM &= 100 + .2(Y) \\I &= 2500 - 1000 r \\G &= 2000 \\X &= 1000\end{aligned}$$

(i) Find the equation for the IS curve

IS is found from  $Y=AD$  identity. Hence, by substituting respective values of  $C$ ,  $I$ ,  $G$ , and  $NX$ , you should be able to find:

$$Y = 5450 + (0.61)Y - 1000 r \quad \text{or} \quad r = 5.45 - (0.00039) Y.$$

(ii) Find the equation for the LM curve

LM is found from  $MS = MD$  identity. Hence, by substituting respective values of  $MD$  and  $MS$  one should be able to find  $r = -0.20 - (0.00002)Y$

(iii) Calculate the equilibrium levels of:  $Y$ ,  $r$ ,  $C$ ,  $I$ ,  $NX$ , and Government Budget

Using IS-LM identity, one should be able to find:

$$Y=13,780; r=0.075 (7.5\%); C=11,212.2; I=2424.4; NX=-1856.1; BS=-121.95$$