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ECON 202
INTERMEDIATE MACROECONOMICS **2 April 2007**
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Exercise VI
Two-Period Models—Partial Equilibrium

1. Suppose that Daniel has income of y_1 when he is young and y_2 when he is old. Initially, the real interest rate is r_1. The utility function of Daniel is $U = \frac{c_1^{1-\theta}}{1-\theta} + \beta \frac{c_2^{1-\theta}}{1-\theta}$, where β is the discount factor.

(i) Find the optimal values of c_1 and c_2.

(ii) Show that $\frac{\partial c_1^*}{\partial y_1} > 0$, $\frac{\partial c_2^*}{\partial y_1} > 0$, and $\frac{\partial s}{\partial y_1} > 0$.

(iii) Show that $\frac{\partial c_1^*}{\partial y_2} > 0$, $\frac{\partial c_2^*}{\partial y_2} > 0$, and $\frac{\partial s}{\partial y_2} < 0$.

(iv) Show that $\frac{\partial c_1^*}{\partial r} = ?$, $\frac{\partial c_2^*}{\partial r} > 0$, and $\frac{\partial s}{\partial r} = ?$ if the consumer is a lender in the current period. (**Difficult**)

(v) Show that $\frac{\partial c_1^*}{\partial r} < 0$, $\frac{\partial c_2^*}{\partial r} = ?$, and $\frac{\partial s}{\partial r} > 0$ if the consumer is a borrower in the current period. (**Difficult**)

2. Suppose that Daniel has income of y_1 = 400 when he is young and y_2=100 when he is old. Initially, the real interest rate is r_1 = 25%. The utility function of Daniel is $U = \frac{c_1^{1-\theta}}{1-\theta} + \beta \frac{c_2^{1-\theta}}{1-\theta}$, where β = 0.8 is the discount factor.

(i) Find the optimal values of c_1, c_2 and $U(\cdot)$ U for $\theta = 0.5$.

(ii) Suppose now that y_1 has been raised to 500. Find the optimal values of c_1 and c_2.

(iii) Suppose now that y_2 has been raised to 150. Find the optimal values of c_1 and c_2.

(iv) Suppose now that r has been raised to 30%. Find the optimal values of c_1 and c_2. Try to disentangle substitution effect from income effect, using the Hicksian compensation.

3. Use the following utility function and budget constraint to answer the given questions.

$$U = \ln(c_1) + \beta \ln(c_2) \text{ s.t. } c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

where c_1 is this period's consumption and c_2 is next period's consumption, y_1 is current income and y_2 is net period's income, and r is the real interest rate.

(i) Determine the MRS (marginal rate of substitution) for this consumer.

(ii) Find the optimal consumption bundle (c_1, c_2) as a function of y_1, y_2, and r for $\beta = 1$. Interpret what $\beta = 1$ does mean. Sketch a graph of this solution.

(iii) Let $y_1 = y_2 = 105$ and r = .05. Use these values to compute current consumption and determine if this consumer is a saver or borrower given this endowment. Show the endowment point on your graph.

(iv) For the same income endowment, determine what happens to c_1 if r increases to r= 0.1. Which effect dominates the current consumption decision: income or substitution effect?

4. Consider the following aggregate consumption function and government budget constraint:

$$C_1 = \frac{1}{2}\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}\right] \text{ and } G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

(i) Show that Ricardian Equivalence holds when consumers recognize that future taxes must cover the current deficit and future spending.

(ii) Since current consumption, C_1, only changes if G_1 or G_2 changes, what happens to private savings (Y-T-C) if current taxes are reduced, with no change in G? (You may assume Y is unchanged.)

(iii) Use the consumption function solved in terms of G_1 and G_2 to answer these questions numerically. Let $Y_1 = Y_2 = 125$ and r = 0.05.

- If $G_1 = G_2 = 20$, what is current consumption, C_1?
- What happens to C_1 if G_1 decreases to 10 with no change in G_2?
- What happens to C_1 if G_1 decreases to 10 and G_2 decreases to 9.5?
- Use your answers above to infer the MPC (marginal propensity to consume) for a temporary increase in income and the MPC for a permanent increase in income. (Hint: refer your textbook to answer this question)