

ECON 202  
INTERMEDIATE MACROECONOMICS  
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**Key to Exercise 05**  
**Two-Period Models—Partial Equilibrium**

1. (**Budget-constraint problem**) Daniel has income of  $y_1 = 400$  when he is young and  $y_2 = 100$  when he is old. Initially, the real interest rate is  $r_1 = 25\%$ . Daniel chooses to consume  $c_1^* = 240$  when he is young.

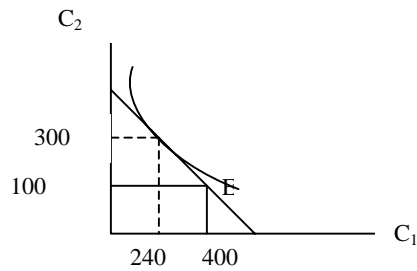
a) What is the present value of Daniel's lifetime resources? What is the future value of Daniel's lifetime resources? How much does Daniel consume when he is old ( $c_2^*$ )? Is Daniel a borrower or a lender?

$$WE = 400 + 100/(1.25) = 480 \rightarrow (1+r) WE = 600$$

b) Illustrate Daniel's consumption/saving choice in a figure. Label your figure carefully with numerical values. (What is on the axes? What is the slope of the budget set? How much could Daniel consume in the present if he planned no consumption in the future? How much could Daniel consume in the future if he consumed nothing in the present? How much could Daniel consume in the present and the future if he planned not to borrow or lend? How much is Daniel actually planning to consume in the present and the future?)

$$c_1^* + \frac{c_2^*}{1+r} = 480 \Rightarrow c_2^* = 300$$

$$s^* = y_1 - c_1^* = 400 - 240 = 160$$



Now suppose that the interest rate increases to  $r_2 = 100\%$ . Daniel's choice under this scenario is to consume  $c_1^{**} = 225$  when he is young.

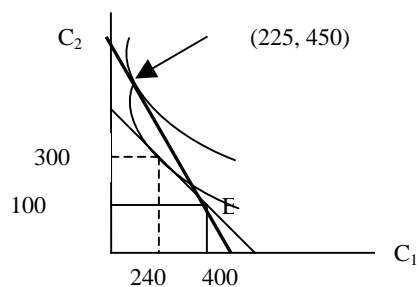
c) How much does Daniel consume when he is old ( $c_2^{**}$ )?

$$225 + \frac{c_2^*}{2} = 400 + \frac{100}{2} \Rightarrow c_2^* = 450$$

As the interest rate increases, we delay our consumption.

$$s^* = y_1 - c_1^* = 400 - 240 = 160$$

d) Redraw your figure from part b), and add to it the illustration of what happens in response to the increase in the real interest rate.



e) Break the change in consumption into an income effect and a substitution effect. In your figure above, label the following points:

- A: no-borrowing no-lending point (E)  
B: original consumption point (240, 300)  
C: new consumption point (225, 450)  
D: consumption choice when there is only a substitution effect.  
(Eliminate the income effect by a Hicks-compensation.) (A point to the left of (240, 300 with the new slope of the constraint))

f) Does the income effect raise or lower current consumption? Does the substitution effect raise or lower current consumption? If the two effects oppose, which one dominates in this example?

Income effect raises current consumption

Substitution effect lowers current consumption

2. (Partial-equilibrium problem) Suppose that Daniel has income of  $y_1 = 400$  when he is young and  $y_2=100$  when he is old. Initially, the real interest rate is  $r_1 = 25\%$ . The utility function of Daniel is  $U = \frac{c_1^{1-\theta}}{1-\theta} + \beta \frac{c_2^{1-\theta}}{1-\theta}$  where  $\beta$  is the discount factor. Find the optimal values of  $c_1$ ,  $c_2$  and  $U(\cdot)$  for  $\theta = 0.5$ ,  $\beta = 0.8$ .

$$L = \frac{c_1^{1-\theta}}{1-\theta} + \beta \frac{c_2^{1-\theta}}{1-\theta} - \lambda \left\{ c_1 + \frac{c_2}{1+r} - y_1 + \frac{y_2}{1+r} \right\} \Rightarrow$$

$$L = \frac{c_1^{0.5}}{0.5} + (0.8) \frac{c_2^{0.5}}{0.5} - \lambda \left\{ c_1 + \frac{c_2}{1.25} - 480 \right\}$$

$$\frac{\partial L}{\partial c_1} = c_1^{-0.5} - \lambda = 0 \quad (1)$$

$$\frac{\partial L}{\partial c_2} = (0.8)c_2^{-0.5} - \frac{\lambda}{1.25} = 0 \quad (2)$$

From (1) and (2), we get  $c_1 = c_2$ . Using this in the two-period intertemporal problem, we find that

$$c_1 = c_2 = 266.67 \text{ and } s = 133.33$$