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Key to Exercise 4
A General Equilibrium Model of Classical Model

1. The following table shows the *labor requirements* in each country:

	Computers	Textiles
Home	1	1
Foreign	3	2

Suppose that labor endowments of Home and Foreign countries are 20 and 80, respectively. Suppose also that preferences are $U(x_C, x_T) = (0.4)\log(x_C) + (0.6)\log(x_T)$ and $U(x_C^*, x_T^*) = (0.4)\log(x_C^*) + (0.6)\log(x_T^*)$, where x_C is consumption of computers and x_T is consumption of textiles.

(i) Find the demand for computer and textile industries for Home and Foreign.
(ii) Find the equilibrium relative price and relative quantity under autarky for Home and Foreign.
(iii) Find the equilibrium output and labor demand for computer and textile industries under autarky for Home and Foreign.

Suppose now that the two countries decide to open their economies to trade. Answer the following questions accordingly.

(iv) Find the relative supply of the World.
(v) Find the relative demand for the World.
(vi) Find the equilibrium relative world price and relative world output.
(vii) Show that both countries gain from trade.
(viii) Show that world gains from trade.

Answers

(i) Demand for computers and textiles can be found by maximizing utility subject to budget constraint. The first step is to set up a Lagrangian:

$$L = (0.4)\log(x_C) + (0.6)\log(x_T) - \lambda\{P_C x_C + P_T x_T - Y\}$$

where λ is Lagrange multiplier, P_C is price of a computer, P_T is price of textile, and Y is income (which is equal to wL). First order conditions are: $\frac{\partial L}{\partial x_C} = (0.4)\frac{1}{x_C} - \lambda P_C = 0$,

$\frac{\partial L}{\partial x_T} = (0.6)\frac{1}{x_T} - \lambda P_T = 0$, $\frac{\partial L}{\partial \lambda} = P_C x_C + P_T x_T - Y = 0$. Using the first and second ones in the third first-order condition, one can easily show that $(x_C)^d = \frac{(0.4)Y}{P_C}$ and $(x_T)^d = \frac{(0.6)Y}{P_T}$.

Though we will not repeat calculations here, we may guess that similar calculations for the Foreign would yield:

$(x_C^*)^d = \frac{(0.4)Y^*}{P_C^*}$ and $(x_T^*)^d = \frac{(0.6)Y^*}{P_T^*}$

(ii) In order to find the equilibrium values of price and quantity, we need to first determine relative demand for and relative supply of computers in terms of textiles. Relative demand is just the ratio of computer demand to textile demand, which is $RD = \frac{(x_C)^d}{(x_T)^d} = \frac{(0.4)}{(0.6)}\frac{1}{P_C / P_T}$. For the Foreign country, it is $RD = \frac{(x_C^*)^d}{(x_T^*)^d} = \frac{(0.4)}{(0.6)}\frac{1}{P_C^* / P_T^*}$.
For finding the relative supply, it is enough to check what the equilibrium process leads to. It is easy to see that equilibrium process implies $P_C = w$ and $P_T = w$, if both industries exist in the market. Remember that a representative firm tends infinite production of computers (and zero textile) if $P_C > w$, and zero production of computers (and infinite production of textile) if $P_C < w$. Hence, $\frac{P_C}{P_T} = 1$. Below this, relative supply of computer is zero, and above this, it is infinite. Hence, given that both commodities are essential in consumption, $\frac{P_C}{P_T} = 1$ must hold. Since relative price is given, it is easy to find out RD from above: $RD = \frac{(0.4)}{(0.6)}\frac{1}{1} \Rightarrow RD = 0.66$.
For the foreign country, calculations are identical. Relative Demand is found as $RD^* = 0.44$ and equilibrium relative price (relative supply) as $\frac{P_C^*}{P_T^*} = \frac{3}{2} = 1.5$.

(iii) In order to find the equilibrium output and labor demand in each industry in each country, we need to go back to demand functions. Since our model is a general equilibrium one, we may replace income in demand by its equivalent. In particular,

$$(x_C)^d = \frac{(0.4)Y}{P_C} = \frac{(0.4)wL}{P_C} = \frac{(0.4)(P_C/1)(20)}{P_C} = 8$$

$$(x_T)^d = \frac{(0.6)Y}{P_T} = \frac{(0.6)wL}{P_T} = \frac{(0.6)(P_T/1)(20)}{P_T} = 12$$

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Labor demand in each industry can be found by noting that $(L_C)^d = 1 \cdot (x_C)^d = 8$ and $(L_T)^d = 1 \cdot (x_T)^d = 12$. Note that $(L_C)^d + (L_T)^d = 20$.
Similar calculations for the Foreign country would yield that

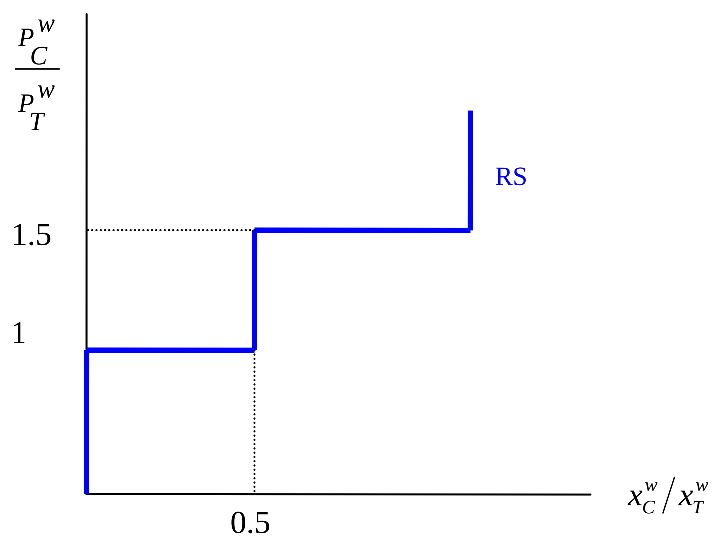
$$(x_C^*)^d = \frac{(0.4)Y^*}{P_C^*} = \frac{(0.4)w^* L^*}{P_C^*} = \frac{(0.4)(P_C^*/3)80}{P_C^*} = 10.67 \text{ and}$$

$$(x_T^*)^d = \frac{(0.6)Y^*}{P_T^*} = \frac{(0.6)w^* L^*}{P_T^*} = \frac{(0.6)(P_T^*/2)80}{P_T^*} = 24$$

can be found. Labor demand is $(L_C^*)^d = 3(x_C^*)^d = 3(10.67) = 32$ and $(L_T^*)^d = 2(x_T^*)^d = 2(24) = 48$, respectively. Note that $(L_C^*)^d + (L_T^*)^d = 32 + 48 = 80$.

(iv) In order to find the relative supply of world, we need to understand how production is determined by (changes in) prices. We may identify 5 possibilities:

(a) If relative price is less than 1, than both countries specialize in production of textiles and hence relative supply of computers become zero.
(b) If relative price is equal to 1, then Home country produces some textiles and some computers and the foreign country remains specialized in textiles.
(c) If relative price is greater than 1 but less than 1.5, then Home country produces only computers and the foreign country remains specialized in textiles.
(d) If relative price is equal to 1.5, then Home country produces only (=specializes) computers and the foreign country produces some combination of computers and textiles.
(e) If relative price is greater than 1.5, then both countries produce only computers.



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(v) World's relative demand $\frac{(x_C^w)^d}{(x_T^w)^d}$ is

$$\frac{(x_C^w)^d}{(x_T^w)^d}=\frac{(x_C)^d+(x_C^*)^d}{(x_T)^d+(x_T^*)^d}=\frac{\frac{(0.4)Y}{P_C}+\frac{(0.4)Y^*}{P_C}}{\frac{(0.6)Y}{P_T}+\frac{(0.6)Y^*}{P_T}}=\frac{\frac{(0.4)}{P_C}(Y+Y^*)}{\frac{(0.6)}{P_T}(Y+Y^*)}=\frac{0.4}{0.6}\frac{1}{P_C^w/P_T^w}$$

Same as closed economy relative demand!

(vi) World's equilibrium price and quantity are either of

(a) $RS=RD\Rightarrow\frac{x_C^w}{x_T^w}=0.67\quad and\quad \frac{P_C^w}{P_T^w}=1$

(b) $RS=RD\Rightarrow\frac{x_C^w}{x_T^w}=0.44\quad and\quad \frac{P_C^w}{P_T^w}=1.5$

(c) $RS=RD\Rightarrow\frac{x_C^w}{x_T^w}=0.5\quad and\quad \frac{P_C^w}{P_T^w}=\frac{4}{3}=1.33$

From the figure above, you may easily see that only (c) is possible.

(vii) Both countries gain from trade. We always measure this by checking the welfare gain:

$$(x_C)^d=\frac{(0.4)Y}{P_C^w}=\frac{(0.4)wL}{P_C^w}=\frac{(0.4)(P_C^w/1)(20)}{P_C^w}=8$$

$$(x_T)^d=\frac{(0.6)Y}{P_T^w}=\frac{(0.6)wL}{P_T^w}=\frac{(0.6)(P_C^w/1)(20)}{P_T^w}=(0.6)(20)\frac{P_C^w}{P_T^w}=(0.6)(20)(1.33)=16$$

Similarly,

$$(x_C^*)^d=\frac{(0.4)Y^*}{P_C^w}=\frac{(0.4)w^*L^*}{P_C^w}=\frac{(0.4)(P_T^w/2)80}{P_C^w}=\frac{(0.4)80}{2(P_C^w/P_T^w)}=\frac{(0.4)80}{2(1.33)}=12 \text{ and}$$

$$(x_T^*)^d=\frac{(0.6)Y^*}{P_T^w}=\frac{(0.6)w^*L^*}{P_T^w}=\frac{(0.6)(P_T^w/2)80}{P_T^w}=24$$

Obviously, Textiles consumption in Home and Computer consumption in Foreign countries are higher than the autarkic case.

(viii) World gain can be shown in terms of consumption and production:

We have already shown that total world consumption has increased:

	Before trade	After trade
Computers	8+10.67=18.67	8+12=20
Textiles	12+24=36	16+24=40

Obviously, world production must follow this increase:

	Before trade	After trade
Computers	8+10.67=18.67	20
Textiles	12+24=36	40