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# Key to Exercise 04
## The Static General Equilibrium Model of Consumption-Leisure Tradeoff

**1**. (**Linear Production Technology**) Suppose that utility function $u$ of a representative agent is $u = c^{\alpha} l^{1-\alpha}$, where $c$ is consumption of physical goods and $l$ is consumption of leisure. Suppose that production technology is represented by $y = zN$ where $y$ is output, $z$ is productivity parameter and $N$ is labor demand. We assume that $h = l + N$ and $w$ is the real wage. There is no government in the economy.

a) Find the optimal values of $c$, $l$, $N$, $y$, $w$, and $u$ under the competitive equilibrium assumption.

$$L = c^{\alpha} l^{1-\alpha} - \lambda\{c + wl - wh - \pi\}$$

$$\frac{\partial L}{\partial c} = \alpha c^{\alpha-1} l^{1-\alpha} - \lambda = 0 \qquad (1)$$

$$\frac{\partial L}{\partial l} = (1-\alpha) c^{\alpha} l^{-\alpha} - \lambda w = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = c + wl - wh - \pi = 0 \qquad (3)$$

From (1) and (2), we get $c = [\alpha/(1-\alpha)]wl$. Since $\pi = 0$ under constant returns to scale technology, it is straightforward to show that $l^* = (1-\alpha)h$ and $c^* = \alpha zh$. Noticeably, $w = z$, which can be easily found from profit maximization process.

b) Find the optimal values of $c$, $l$, $N$, $y$, and $u$ under the social planner's solution assumption. Are the results different? Why or why not?

In this case, our static maximization problem reads

$$L = c^{\alpha} l^{1-\alpha} - \lambda\{c - z(h - l)\}$$

The rest of the solution program is similar and the results obtained are identical.

c) Find the impact of one-time permanent changes in exogenous variables on endogenous variables in the model.

| | z | h |
|---|---|---|
| c | + | + |
| w | + | 0 |
| y | + | + |
| N | 0 | + |
| l | 0 | + |

**2.** (**Cobb-Douglas production technology**) Suppose that utility function $u$ of a representative agent is $u = c^{\alpha} l^{1-\alpha}$, where $c$ is consumption of physical goods and $l$ is consumption of leisure. Suppose that production technology is represented by $y = zK^{\beta} N^{1-\beta}$ where $z$ is productivity parameter, $K$ is a given amount of physical capital stock, and $N$ is labor demand. We assume that $h = l + N$, $w$ is the real wage, and $\pi$ is profits. There is no government in the economy.

a) Find the optimal values of $c$, $l$, $N$, $y$, $w$, $\pi$, and $u$ under the competitive equilibrium assumption.

Since K is constant (in the short run), profits are positive. Hence, we need to find out profits for consumer maximization.

$$\pi = z\overline{K}^{\beta} N^{1-\beta} - wN \Rightarrow$$

$$\frac{\partial \pi}{\partial N} = (1-\beta) z\overline{K}^{\beta} N^{-\beta} - w = 0 \Rightarrow$$

$N^d = \left( \frac{(1-\beta) z\overline{K}^{\beta}}{w} \right)^{1/\beta}$ and $\pi = (\beta/(1-\beta))wN$. Next, we turn to consumer maximization problem:

$$L = c^{\alpha} l^{1-\alpha} - \lambda \{ c + wl - wh - \pi \}$$

$$\frac{\partial L}{\partial c} = \alpha c^{\alpha-1} l^{1-\alpha} - \lambda = 0 \qquad (1)$$

$$\frac{\partial L}{\partial l} = (1-\alpha) c^{\alpha} l^{-\alpha} - \lambda w = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = c + wl - wh - \pi = 0 \qquad (3)$$

From (1) and (2), we get $c = [\alpha/(1-\alpha)]wl$ and $N^S = \alpha h - [(1-\alpha)\beta/(1-\beta)] \left( \frac{(1-\beta) z\overline{K}^{\beta}}{w} \right)^{1/\beta}$. From $N^d = N^s$ equality, we can find that $w^* = z\overline{K}^{\beta}(1-\beta)^{1-\beta}\left(\frac{1-\alpha\beta}{\alpha h}\right)^{\beta}$. It is straightforward to show that $l^* = \frac{(1-\alpha)h}{1-\alpha\beta}$, $c^* = z\overline{K}^{\beta}\left(\frac{(1-\beta)\alpha h}{1-\alpha\beta}\right)^{1-\beta}$, and $\pi^* = \beta z\overline{K}^{\beta}\left(\frac{(1-\beta)\alpha h}{1-\alpha\beta}\right)^{1-\beta}$.

b) Find the optimal values of $c$, $l$, $N$, $y$, and $u$ under the social planner's solution assumption. Are the results different? Why or why not?

In this case, our static maximization problem reads

$$L = c^{\alpha} l^{1-\alpha} - \lambda\{c - zK^{\beta}(h-l)^{1-\beta})\}$$

The rest of the solution program is similar and the results obtained are identical.

c) Find the impact of one-time permanent changes in exogenous variables on endogenous variables in the model.

| | z | h | K |
|---|---|---|---|
| c | + | + | + |
| w | + | - | + |
| y | + | + | + |
| N | 0 | + | 0 |
| l | 0 | + | 0 |

**3**. (**With Government sector**) Suppose that utility function $u$ of a representative agent is $u = c^{\alpha} l^{1-\alpha}$, where $c$ is consumption of physical goods and $l$ is consumption of leisure. Suppose that production technology is represented by $y = zK^{\alpha} N^{1-\alpha}$ where $z$ is productivity parameter, $K$ is a given amount of physical capital stock, and $N$ is labor demand. We assume that $h = l + N$ and $w$ is the real wage. We also assume that there is a government in the economy that charges lump-sum taxes on profits, which are spent on exogenously determined government expenditures, $G$.

a) Try to find the optimal values of $c$, $l$, $N$, $y$, $w$, and $u$ under the competitive equilibrium assumption.

Since K is constant (in the short run), profits are positive. Hence, we need to find out profits for consumer maximization.

$$\pi = z\overline{K}^{\alpha} N^{1-\alpha} - wN \Rightarrow$$

$$\frac{\partial \pi}{\partial N} = (1-\alpha) z\overline{K}^{\alpha} N^{-\alpha} - w = 0 \Rightarrow$$

$N^d = \left(\frac{(1-\alpha)z\bar{K}^\alpha}{w}\right)^{1/\alpha}$ and $\pi = (\alpha/(1-\alpha))wN$. Next, we turn to consumer maximization problem:

$$L = c^\alpha l^{1-\alpha} - \lambda\{c + wl - wh - (\pi - T)\}$$

$$\frac{\partial L}{\partial c} = \alpha c^{\alpha-1} l^{1-\alpha} - \lambda = 0 \qquad (1)$$

$$\frac{\partial L}{\partial l} = (1-\alpha)c^\alpha l^{-\alpha} - \lambda w = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = c + wl - wh - (\pi - T) = 0 \qquad (3)$$

From (1) and (2), we get $c = [\alpha/(1-\alpha)]wl$ and $N^S = \alpha h - [(1-\alpha)/w)](\pi - G)$. Unfortunately, we cannot solve the problem from the $N^d = N^s$ equality this time. However, by using implicit differentiation in the following equation,

$$\alpha h w^{1/\alpha} + (1-\alpha)G w^{(1-\alpha)/\alpha} = \left((1-\alpha)zK^\alpha\right)^{1/\alpha}(1+\alpha),$$

it is straightforward to show that $\frac{dw}{dG} < 0$, $\frac{dw}{dz} > 0$, $\frac{dl}{dG} < 0$, $\frac{dN}{dG} > 0$, etc.

b) Try to find the optimal values of $c$, $l$, $N$, $y$, and $u$ under the social planner's solution assumption. Are the results different? Why or why not?

In this case, our static maximization problem reads
$L = c^\alpha l^{1-\alpha} - \lambda\{c + G - zK^\alpha(h-l)^{1-\alpha})\}$
The rest of the solution program is similar. It is not possible to find closed-form solution to the problem.

c) Find the impact of one-time permanent changes in exogenous variables on endogenous variables in the model.

See a.

**4**. (**With externality**) Suppose that utility function $u$ of a representative agent is $u = c^\alpha l^{1-\alpha}$, where $c$ is consumption of physical goods and $l$ is consumption of leisure. Suppose that production technology is represented by $y = zK^\beta N^{1-\beta} N^\gamma$ where $z$ is productivity parameter, $K$ is a given amount of physical capital stock, $N$ is labor demand, and $\gamma$ is a parameter that determines the extent of externality in the economy.

We assume that $\gamma > 0$, that is, the stock of labor has a positive externality effect on production. We also assume that $h = l + N$ and $w$ is the real wage. There is no government in the model.

a) Find the optimal values of $c$, $l$, $N$, $y$, $w$, and $u$ under the competitive equilibrium assumption.

There is externality this time. In market solution, agents are not aware of this positive externality. Hence, profit maximization goes as follows:

$$\pi = z\overline{K}^{\beta} N^{1-\beta} N^{\gamma} - wN \Rightarrow$$

$$\frac{\partial \pi}{\partial N} = (1-\beta) z\overline{K}^{\beta} N^{-\beta+\gamma} - w = 0 \Rightarrow$$

$$N^d = \left( \frac{(1-\beta) z\overline{K}^{\beta}}{w} \right)^{1/(\beta-\gamma)}$$ and $\pi = (\beta/(1-\beta))wN$. Next, we turn to consumer maximization problem:

$$L = c^{\alpha} l^{1-\alpha} - \lambda\{c + wl - wh - \pi\}$$

$$\frac{\partial L}{\partial c} = \alpha c^{\alpha-1} l^{1-\alpha} - \lambda = 0 \qquad (1)$$

$$\frac{\partial L}{\partial l} = (1-\alpha) c^{\alpha} l^{-\alpha} - \lambda w = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = c + wl - wh - \pi = 0 \qquad (3)$$

From (1) and (2), we get $c = [\alpha/(1-\alpha)]wl$ and $N^S = \alpha h - (1-\alpha)\pi / w$. From $N^d = N^s$ equality, we can find that $w^* = z\overline{K}^{\beta}(1-\beta)^{1-\beta+\gamma}\left(\frac{\alpha h}{1-\alpha\beta}\right)^{-\beta+\gamma}$. It is straightforward to show that $l^* = \frac{(1-\alpha)h}{1-\alpha\beta}$, $c^* = z\overline{K}^{\beta}\left(\frac{(1-\beta)\alpha h}{1-\alpha\beta}\right)^{1-\beta+\gamma}$, etc.

b) Find the optimal values of $c$, $l$, $N$, $y$, and $u$ under the social planner's solution assumption. Are the results different? Why or why not?

In this case, our static maximization problem reads

$$L = c^{\alpha} l^{1-\alpha} - \lambda\{c - zK^{\alpha}(h-l)^{1-\alpha+\gamma})\}$$

The rest of the solution program is similar. But results change due to the fact that social planner is aware of the externality and it takes it into account.

This is reflected in the solution. For example, let us take consumption:

$c^{**} = z\overline{K}^{\beta}\left(\frac{(1-\beta+\gamma)\alpha h}{1-\alpha\beta+\alpha\gamma}\right)^{1-\beta+\gamma}$. It is expected that $u^{**} > u^{*}$.

c) Compare this model with the previous model (i.e., question #3) and state the qualitative difference between the two models?

The difference is the fact that social planner takes the externality into account.